

M2 Internship: Mixing time of structured dense graphs

Supervisors: Malory Marin (LIP), Théo Pierron (LIRIS), Rémi Watrigant (LIP)

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The objective of this internship is motivated by the following practical scenario: a collection of wireless access points (e.g., Wi-Fi) can be modeled as the intersection graph of disks in the plane (called disk graphs), where two disks intersect if their communications interfere. In such a case, they cannot transmit simultaneously on the same frequency. Consequently, the set of access points that can communicate at a given time forms an independent set of the interference graph. This independent set evolves continuously depending on the traffic that each access point needs to send or receive.

This phenomenon can be accurately modeled by the so-called Glauber dynamics on independent sets. In a nutshell, at each step, a vertex is selected uniformly at random and may “enter” or “leave” the current independent set with a given probability. When the process stabilizes, the performance of each access point (its throughput) can be inferred from the fraction of independent sets it was part of. The number of steps required for convergence (as a function of the number of vertices) is known as the mixing time of the dynamics.

Understanding the boundary between polynomial and exponential mixing times for various graph classes is a classical research topic. Sparse graph classes (e.g., bounded-degree graphs, planar graphs, or graphs of bounded treewidth) are known to exhibit small mixing times. For dense classes, the situation is less clear: for example, complete bipartite graphs have exponential mixing time, whereas complete graphs do not.

The latter remark is rather fortunate, since disk graphs may contain large complete graphs but no complete bipartite graphs—a property shared by many geometric intersection graph classes. Moreover, they go beyond this “safety test”: they were recently proven to have subexponential mixing time.

The internship will focus on obtaining similar results using recently introduced graph parameters that capture the structural properties most relevant to mixing time, going beyond sparse graph classes (e.g. tree-independence number, or appropriate weighted tree decompositions). It will also study the links between mixing times and independent set reconfiguration, a closely related topic where one asks whether two independent sets can be transformed to each other through a sequence of atomic changes (that is, modification of a single vertex at a time). For instance, the associated techniques might be of interest for establishing matching lower bounds for the mixing time.

It should be noted that although the subject has a practical motivation, the expected work will mostly be theoretical. Depending on the candidate's profile, the internship may focus on different aspects of theoretical computer science, such as algorithms, computational complexity, graph theory, probabilities, etc.