Laboratoire de mécanique des fluides et d'acoustique

Fluides Complexes et Transferts

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Turbulent transition at the entrance of a plane channel

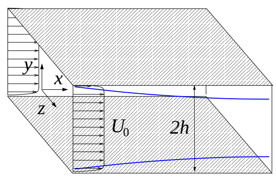
Motivation

The objective is the study of by-pass turbulent transition occurring in a plane channel flow, downstream the entry zone, and more particularly the effect of inlet turbulence and wall roughness. Although the present investigation is currently limited to incompressible flows, one objective is to better understand appearance of cavitation in fuel injector devices (project NadiaBio Mov'eo 2008-2011).

By-pass transition in boudary layers

Transition in boundary layers exposed to free-stream turbulent intensity of order $\geq 1\%$ or more can occur without the mediation of viscous Tollmien-Schlichting instability waves (by-pass transition). Experiments or numerical simulations show the presence in the boundary layers of large, elongated spanwise modulations of the streamwise velocity called streamwise streaks. These disturbances grow in the streamwise direction and are subjected to an instability process leading finally to turbulence breakdown. The physical process explaining emergence of streaks is known as the lift-up effect. It is understood as the result of interaction between streamwise vorticity and the boundary layer shear: streamwise vorticity pushes low momentum fluid away from the wall and high momentum fluid towards the wall and the spanwise modulation obtained in this way is stretched downstream by the mean shear. Streaks can be found as solutions of the linearised stability equations (Orr-Sommerfeld-Squire equations). However, the effect of viscosity eventually dominates these linear solutions, making the growth of the streaks only transient. Stricto sensu, streaks are thus stable perturbations (sub-critical), but their amplitude can reach such large values (30% of the mean flow) than they escape the linear regime and becomes sensitive to secondary perturbations ([1],[2]).

Flow configuration

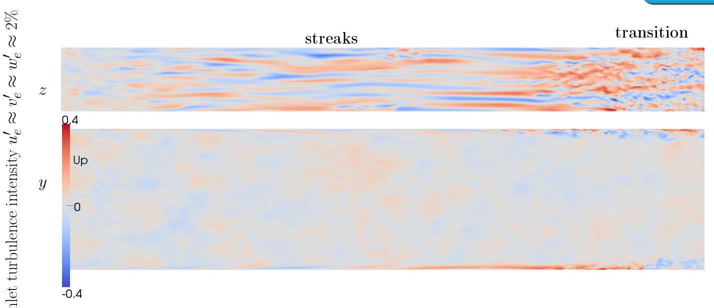


$$Re_h = \frac{U_0 h}{\nu} = 20000$$

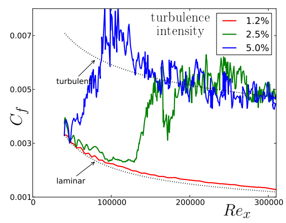
$$40000 \leq Re_x = \frac{U_0 x}{\nu} \leq 320000$$

$\delta_0^* = .017h$: boundary layer displacement at inlet

By-pass transition induced by free-stream turbulence



Streamwise velocity perturbation at distance $2\delta_0^*$ from the wall (top) and in a vertical plane (bottom).



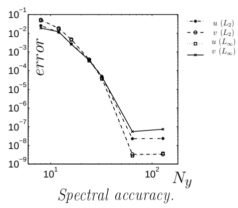
Friction coefficient C_f : transition from Blasius solution to turbulent regime for high enough inlet turbulence intensity.

Numerical method

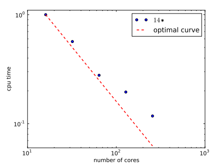
NadiaSpectral

Using the orthogonal decomposition of the velocity field, an efficient Galerkin spectral code has been developed [3] for the analysis of transition in wall-bounded shear flows. In order to use Fourier expansions in the streamwise direction, the solution is forced towards periodicity using the fringe method [4]. The approximation uses Fourier expansions in two directions and the Chebyshev basis proposed by Moser et al. [5] in the third direction in order to satisfy the wall boundary conditions. Using Crank-Nicholson/Adams Bashford time integration, the numerical scheme requires the solution of two 1D sparse linear systems for each Fourier component of the velocity. The method has been implemented in C++ in the NadiaSpectral code and parallelized using MPI and OpenMP.

www.ufrmeca.univ-lyon1.fr/∼buffat/NadiaSpectral



Spectral accuracy.



Parallel efficiency.

Orthogonal decomposition of solenoidal fields

General result *Let be* $\mathbf{y}$ *a unit vector in* $\mathbb{R}^3$ *(the 'vertical' direction) and* $\mathbf{u}$ *a solenoidal field defined in a bounded domain* $\Omega \subset \mathbb{R}^3$, $\mathbf{u}$ *can be decomposed into the sum of two solenoidal,* $L_2(\Omega)$*-orthogonal fields :* $\mathbf{u} = \mathbf{u}_{sq} + \mathbf{u}_{os}$

- $\mathbf{u}_{sq}$ *is 2D, solenoidal, has zero vertical velocity and is determined by the values of* ω*, the vertical vorticity of* $\mathbf{u}$*,*
- $\mathbf{u}_{os}$ *is 3D, solenoidal, has zero vertical vorticity and is determined by the values of* v*, the vertical component of* $\mathbf{u}$ *and by the values of the normal velocity , on the boundary* $\partial\Omega$*.*

For doubly periodic fields in the two directions x and z normal to $\mathbf{y}$: Fourier components can be expressed in function of the components of v and ω. If $(.)^{\alpha\beta}$ denotes the component of wavevector (α, β) and $k^2 = \alpha^2 + \beta^2$:

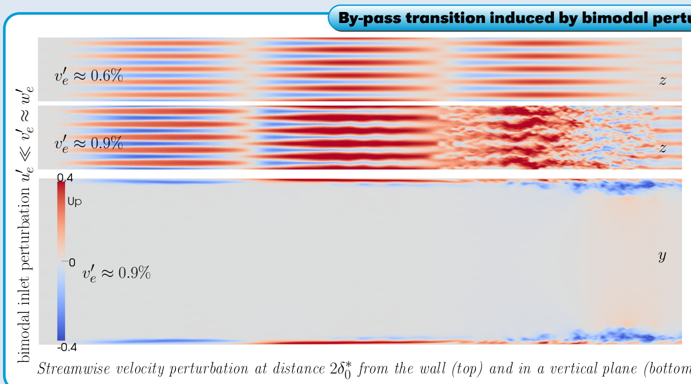
$$\mathbf{u}_{sq}^{\alpha\beta} = \left(-\imath\frac{\beta}{k^2}\omega^{\alpha\beta}, 0, \imath\frac{\alpha}{k^2}\omega^{\alpha\beta}\right) ; \ \mathbf{u}_{os}^{\alpha\beta} = \left(\imath\frac{\alpha}{k^2}\partial_y v^{\alpha\beta}, v^{\alpha\beta}, \imath\frac{\beta}{k^2}\partial_y v^{\alpha\beta}\right)$$

For plane, parallel, shear flows, this decomposition is related to the Orr-Sommerfeld and Squire modal decomposition of the linear stability theory [1]. Using the orthogonal decomposition, the projection of the Navier-Stokes equations linearized around the basis velocity field $(U_b(y), 0, 0)$ reads :

$$\underbrace{\begin{pmatrix} \mathcal{L} & 0 \\ 0 & \mathcal{L} \end{pmatrix} \begin{pmatrix} \mathbf{u}_{os}^{\alpha\beta} \\ \mathbf{u}_{sq}^{\alpha\beta} \end{pmatrix}}_{transport\ and\ viscous\ terms} + \underbrace{\begin{pmatrix} \frac{\imath\alpha}{k^2}\partial_y\left(v^{\alpha\beta}U_b'\right) \\ \frac{\imath\beta}{k^2}\left(v^{\alpha\beta}U_b'\right) \end{pmatrix}}_{interaction\ with\ U_b} = \underbrace{\mathcal{NL}(\mathbf{u}_{os}, \mathbf{u}_{sq})}_{non-linear\ interaction}$$

The linear part of these two scalar equations is equivalent to the classical Orr-Sommerfeld-Squire system [1].

By-pass transition induced by bimodal perturbations : (α, β) (primary) + (α', β') (secondary), $\alpha \ll \beta$



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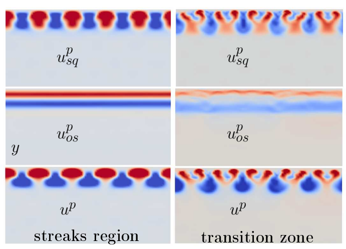
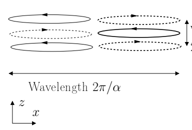


Decomposition of the perturbation in vertical planes.

$$u^p = u_{os}^p + u_{sq}^p$$

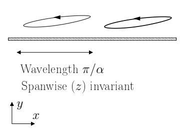
Non linear structure of periodic streaks

In (x, z) plane :



$\mathbf{u}_{sq}^p$
linear response
Mode (α, β)

In (x, y) plane :



$\mathbf{u}_{os}^p$
non-linear response
Modes
$(0, 0) + (2\alpha, 0) + \ldots$

References

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3 Buffat M., Le Penven L., & Cadiou A., *An efficient spectral method based on an orthogonal decomposition of the velocity for transition analysis in wall bounded flow.* Submitted to Computers & Fluids

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5 Moser R. D., Moin P. & Leonard A., *A spectral numerical method for the Navier-Stokes equations with applications to Taylor-Couette flow.* J. Comp. Phys., 1983