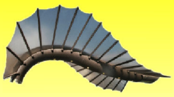

NadiaSpectral: a powerful tool to study transition

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Objectives

DNS of incompressible flows with solid wall interactions (see [1-6])

- boundary layer
- channel flow
- turbulence, by-pass transition
- flows with solid particles
- complex boundaries
- smooth or rough walls

Characteristics of NadiaSpectral

- Galerkin spectral approximation: (Fourier/Chebyshev) ([1])
- based on an **orthogonal decomposition** of solenoidal space ([7])
- **new methods (1)-(2)** to take into account complex boundaries ([2,3,5])
- **efficient parallel implementation (3)** ([4])

DNS of by-pass transition

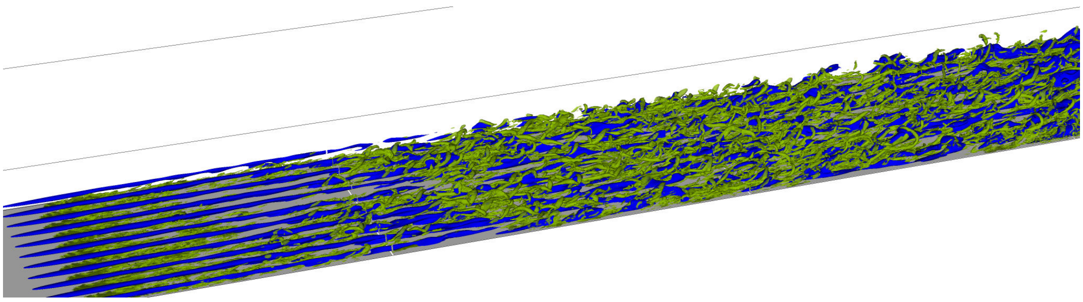

FIGURE 1: Simulation of streaks instabilities at the entrance of a plane channel: Λ_2 criterion (green) and negative iso-pressure (blue)

- at the inlet: optimal mode u'_{opt} (3%) with $\alpha = 0$, $\beta \approx 2/\delta_0 \rightsquigarrow$ strong transient growth $\rightsquigarrow$ generation of streaks
- low-level perturbation ($< 10^{-3} u'_{opt}$) of the optimal mode $\rightsquigarrow$ sinuous and varicose instabilities of the streaks and transition to turbulence

(1) new Fictitious Domain Formulation

Spectral accurate Fictitious Domain Method with Internal Forcing (FDMIF)(see [2],[3])

- Initial problem in a domain Ω_p with complex boundary γ

$$-\Delta u = 0 \quad \text{in } \Omega_p \quad \text{with b.c. } u_{|\gamma} = u_0 \tag{1}$$

- Fictitious domain formulation in a simpler domain Ω

$$\begin{array}{c} -\Delta u = f \quad \text{in } \Omega \\ \text{with } f \text{ such that } f(\boldsymbol{x}) = 0 \text{ if } \boldsymbol{x} \in \Omega_p \text{ and } u_{|\gamma} = u_0 \end{array} \tag{2}$$

- Smooth forcing inside the fictitious domain $\omega = \Omega \backslash \Omega_p$

$$f(\boldsymbol{x}) = \int_{\tilde{\gamma}} \lambda(\boldsymbol{s}) h_\rho(\boldsymbol{x} - \boldsymbol{s})\, d\sigma$$

- λ Lagrange multipliers, h_ρ mollifiers (infinitely differentiable bump)

$$h_\rho(\boldsymbol{x}) = b_\rho(\|\mathbf{x}\|) \quad \text{with } b_\rho(r) = e^{\frac{1}{1-(\frac{\rho}{r})^2}} \text{ for } r \le \rho$$

Periodic Stokes flow over cylinder

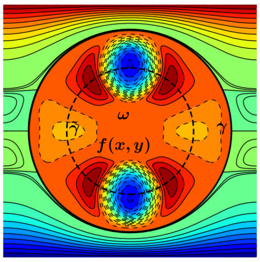

FIGURE 2: **FDMIF** solution in Ω_p with $\boldsymbol{u}_{|\gamma} = 0$ and forcing $\boldsymbol{f(x,y)}$ in the disk ω

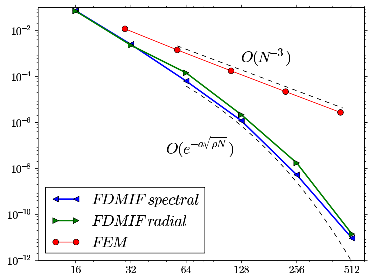

FIGURE 3: L_2 norm in $\Omega \backslash \bar{\omega}$ of the relative error on the velocity versus the number of modes N

(2) Implicit Volume penalisation

Implicit Volume penalisation with a Galerkin spectral formulation in divergence-free space (see [5])

- Initial problem in a domain Ω_p with complex wall boundary γ

$$\frac{\partial \boldsymbol{u}}{\partial t} + \mathcal{NS}(\boldsymbol{u}) = 0 \quad \text{in } \Omega_p \quad \text{with b.c. } \boldsymbol{u}_{|\gamma} = \boldsymbol{u}_0 \tag{3}$$

- Volume penalisation inside ω with $\partial\omega = \gamma$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \mathcal{NS}(\boldsymbol{u}) = \frac{\chi_\omega}{\eta}(\boldsymbol{u} - \boldsymbol{u}_0) \quad \text{in } \Omega = \Omega_p \cup \omega \tag{4}$$

- Implicit integration using operator splitting

$$\begin{aligned} \frac{\boldsymbol{u}^{n+1/2} - \boldsymbol{u}^n}{\Delta t} + \mathcal{NS}(\boldsymbol{u}^n) &= \frac{\chi_\omega}{\eta}(\boldsymbol{u}^{n+1/2} - \boldsymbol{u}_0) \\ \frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^{n+1/2}}{\Delta t} + \mathcal{NS}(\boldsymbol{u}^{n+1}) &= \frac{\chi_\omega}{\eta}(\boldsymbol{u}^{n+1/2} - \boldsymbol{u}_0) \end{aligned}$$

linearly unconditionally stable and $\nabla.\boldsymbol{u}^{n+1} = 0$

Boundary layer over rough wall

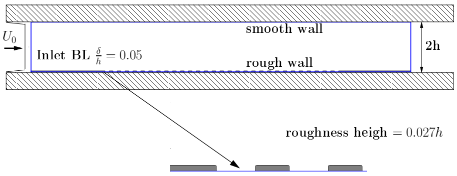

FIGURE 4: Configuration and computational domain

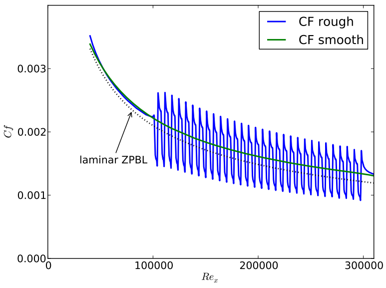

FIGURE 5: Friction coefficient CF over rough and smooth wall

(3) Parallel implementation

Development of efficient numerical tools allowing accurate numerical simulations (see [4])

- modern programming tools (C++, cmake, parallel python, git ..)
- hybrid parallelism (MPI + high level multi-threading)
- optimized parallel decomposition
- scale on Peta-flops parallel computers ($O(10^5)$ cores)
- parallel IO and visualisation (ParaView, VisIt)
- accurate parallel analysis of the results using python scripts

Parallel efficiency

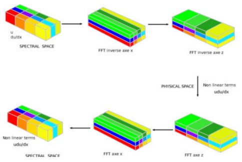

FIGURE 6: Parallel FFT using domain partitioning

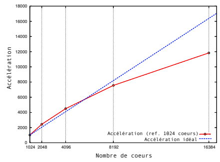

FIGURE 7: Speed up on IBM Blue Gene

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