Solving canonical PDEs using the mimetic finite difference method

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(Le porteur s'engage à participer et présenter son projet aux journées de la GI-EIF)
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Laboratoire : LMFA
Composante : Mécanique
Nature du financement demandé : Stage de M2
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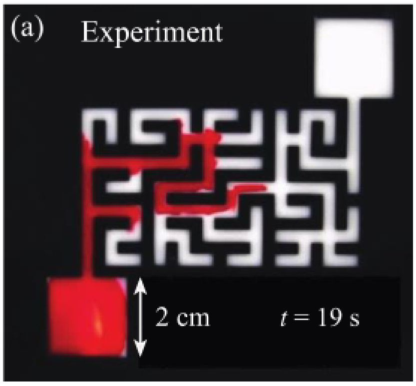

Sujet développé :
This research project focuses on the numerical modelling and simulation of transport phenomena using the mimetic finite difference (MFD) method, a relatively recent approach designed to preserve the fundamental geometric and physical properties of differential operators at the discrete level. Unlike classical finite difference or finite element methods, MFD schemes aim to "mimic" the integral identities of vector calculus—such as divergence, gradient, and curl—ensuring that conservation laws and symmetries remain valid after discretization. The student will investigate how canonical partial differential equations governing transport phenomena (e.g., diffusion, advection, and Poisson-type equations) can be discretized using MFD on general, possibly non-orthogonal or unstructured meshes. The core objective is to formulate and implement these discrete operators, represent them through incidence matrices, and perform simulations on complex geometries. Comparative studies will then be carried out against standard finite difference schemes to assess numerical accuracy, convergence, and computational efficiency. As an example, we have simulated using the MFD a model transport equation for surfactants transported through a liquid maze through the Marangoni effect (see picture above and and our award winning video, Temprano-
Coleto et al. Phys. Rev. Fluids, 2018). We find that the MFD offers advantages for scaling up this time-dependent transport equation across mazes of a much larger scale. However, we have not performed a thorough comparison of its numerical efficiency with respect to other numerical methods. This could be a starting point of the project.
The project will combine theoretical analysis with practical implementation, offering a solid introduction to modern numerical geometry and high-fidelity simulation methods. It is particularly suited for students with a background in applied mathematics or fluid mechanics who are eager to explore advanced discretization techniques for continuum physics.