Finding a Sparse k-Subgraph in Restricted Graph Classes

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Contents

Sparsest k-Subgraph Problem (SkS)

Input: a simple undirected graph $G = (V, E)$, $k \leq |V|$.
Output: a set $S \subseteq V$ of size exactly k.
Goal: minimize $E(S)$ (the number of edges induced by S)

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- Related problems:
  - maximisation version: Densest k-Subgraph (DkS)
    - exact result for DkS on $\mathcal{C}$ $\Leftrightarrow$ exact result for SkS on $\bar{\mathcal{C}}$
  - dual version: Maximum Vertex Coverage (MVC)
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- we study SkS in classes where INDEPENDENT SET is polynomial-time solvable e.g. perfect graphs and their subclasses

In restricted graphs classes:

Densest k-Subgraph

Sparsest k-Subgraph

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- Chordal graphs:
  NP-hard [Corneil,Perl,1984]
  3-approx. [Liazi,Milis,Zissimopoulos,2008]
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  Polynomial
- Planar graphs:
  NP-h (from Independent Set)
  FPT ("obvious" linear kernel)

In this talk:

- *PTAS* in Proper Interval graphs.
- *FPT* algorithm in Interval graphs parameterized by the number of edges in the solution (stronger parameterization than by k).

Definitions

Polynomial-Time Approximation Scheme (PTAS)

A *PTAS* for a minimization problem is an algorithm $\mathcal{A}_\epsilon$ such that for any fixed $\epsilon > 0$, $\mathcal{A}_\epsilon$ runs in polynomial time and outputs a solution of cost $< (1 + \epsilon)OPT$

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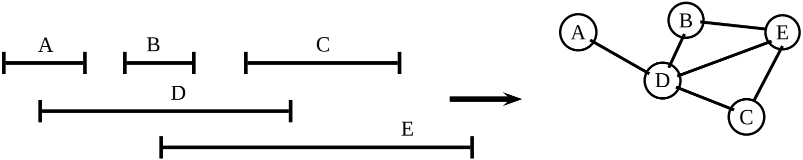
Fixed-Parameter Tractable (FPT)

An *FPT* algorithm for a parameterized problem is an algorithm that exactly solves the problem in $O(f(k).poly(n))$ where n is the size of the instance and k the parameter of the instance.

Definitions

Interval graphs = intersection graphs of intervals on the real line.



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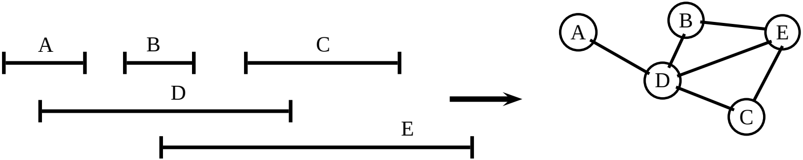

proper interval graph = no interval contains properly another one = unit interval graphs

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PTAS in Proper Interval Graphs

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PTAS in Proper Interval Graphs

Idea of the algorithm:

- sorting intervals according to their right endpoints
- greedy decomposition of the graph into a path of separators
- re-structuration of an optimal solution into a near optimal solution such that all near optimal solutions can be enumerated in polynomial time
- dynamic programming processes the graph through the decomposition, enumerating all possible solutions.

PTAS in Proper Interval Graphs

The decomposition:

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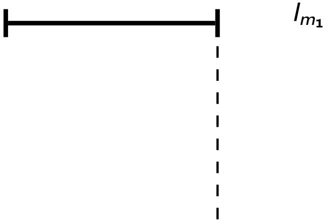

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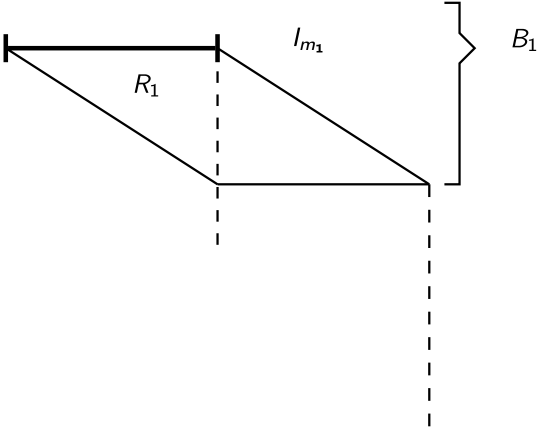

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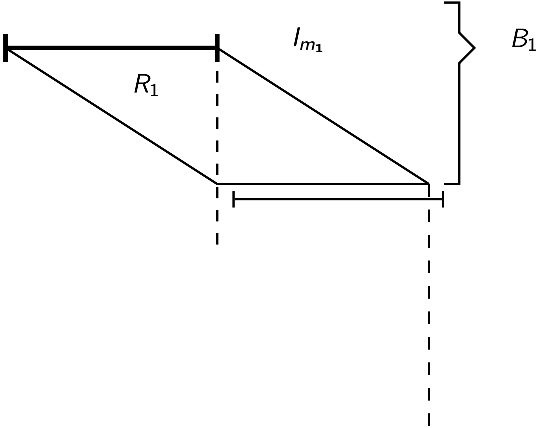

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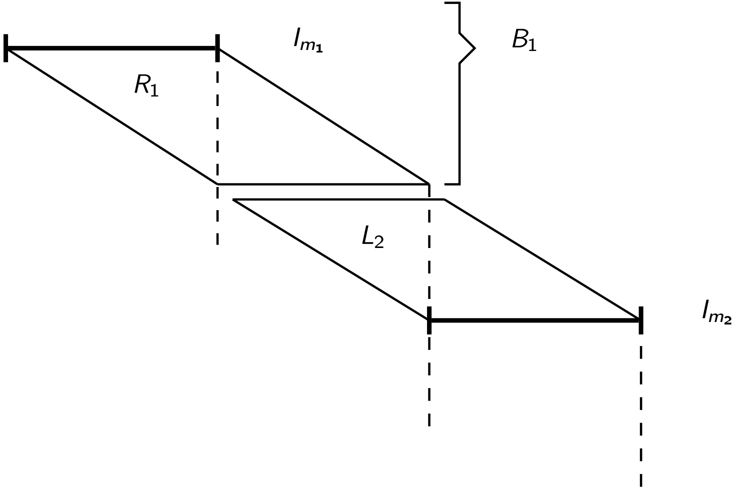

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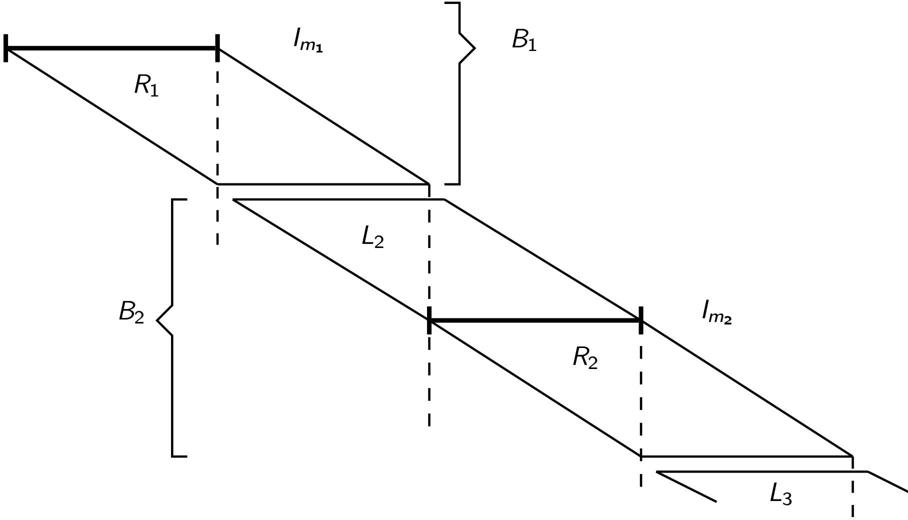

PTAS in Proper Interval Graphs

The decomposition

Remark

The only edges between blocks B_i and B_{i+1} are between R_i and L_{i+1}.
Given $S \subseteq \mathcal{I}$ we have:

$$E(S) = \sum_{i=1}^{a} E(B_i \cap S) + \sum_{i=1}^{a-1} E(R_i \cap S, L_{i+1} \cap S)$$

PTAS in Proper Interval Graphs

Re-structuration of optimal solutions

Compaction

Let $S \subseteq \mathcal{I}$ be a solution, and $S^c = comp(S) \subseteq \mathcal{I}$ such that for each block $i \in \{1, ..., a\}$:

- for all $I \in S \cap L_i$, $comp(I) \in L_i$ and is at the right of I
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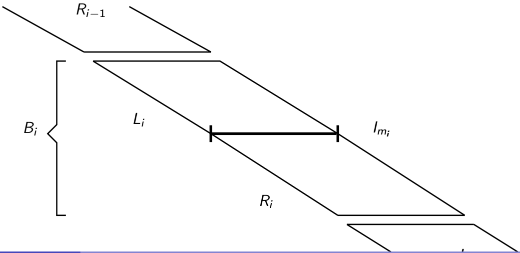

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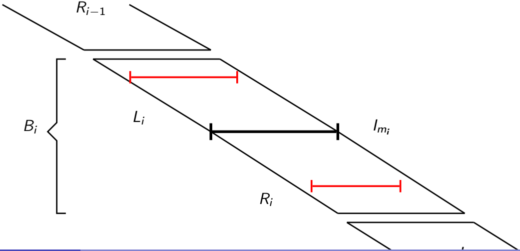

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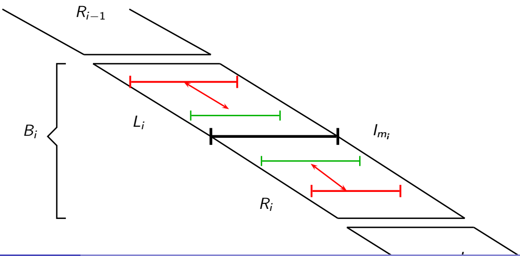

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Lemma

If *comp* is a compaction of a solution S such that for all block $i \in \{1, ..., a\}$, we have

$$E(comp(S \cap B_i)) \leq \rho E(S \cap B_i)$$

Then $comp(S)$ is a ρ-approximation of S.

PTAS in Proper Interval Graphs

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Let us built a compaction that yields a $(1 + \frac{4}{P})$-approximation for any fixed P.

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- we divide X_L into P consecutive subsets of same size $q_L \rightarrow X_1^L, ..., X_P^L$
- we divide X_R into P consecutive subsets of same size $q_R \rightarrow X_1^R, ..., X_P^R$

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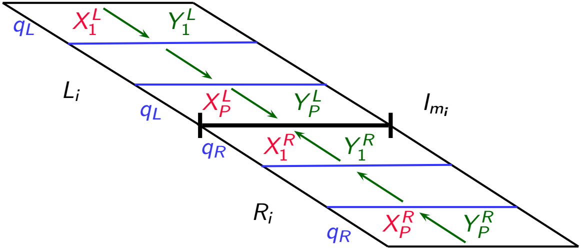
Then define the compaction: for any $t \in \{1, ..., P\}$

- Y_t^L are the q_L rightmost intervals at the left of the rightmost interval of X_t^L
- Y_t^R are the q_R leftmost intervals at the right of the leftmost interval of X_t^R



PTAS in Proper Interval Graphs

Re-structuration of optimal solutions

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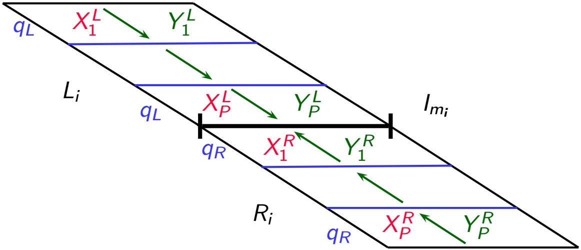


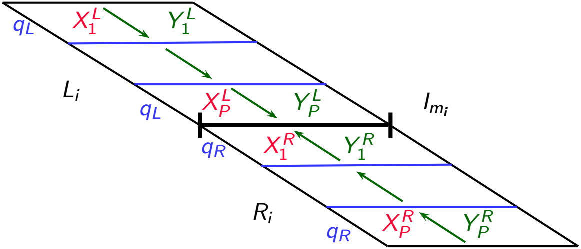
PTAS in Proper Interval Graphs

Re-structuration of optimal solutions

What do we need to construct such a solution ?

- the leftmost interval of X_t^L for $t \in \{1, ..., P\}$
- the rightmost interval of X_t^R for $t \in \{1, ..., P\}$
- x_R, x_L (plus remainders of divisions by P...)

$\Rightarrow 2P + O(1)$ variables ranging in $\{0, ..., n\}$



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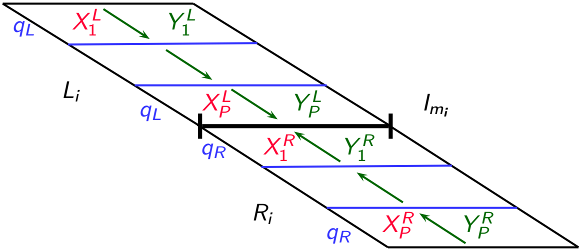

Sketch of proof of the $(1 + \frac{4}{P})$ approximation ratio:

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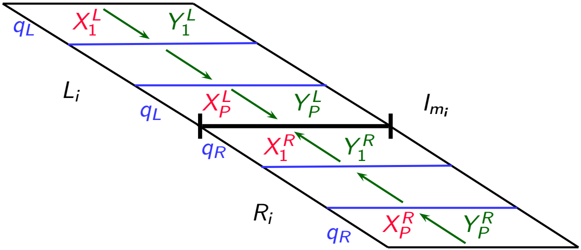

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- $OPT = \binom{x_L}{2} + \binom{x_R}{2} + \sum_{t=1}^{a} \sum_{u=1}^{a} E(X_t^L, X_u^R)$
- $SOL = \binom{x_L}{2} + \binom{x_R}{2} + \sum_{t=1}^{a} \sum_{u=1}^{a} E(Y_t^L, Y_u^R)$

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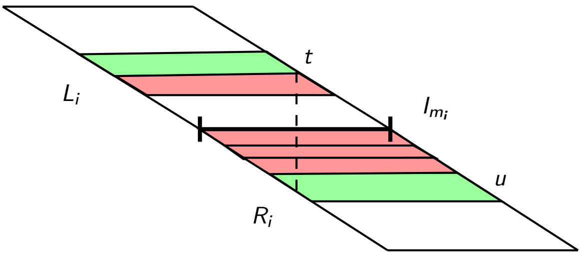

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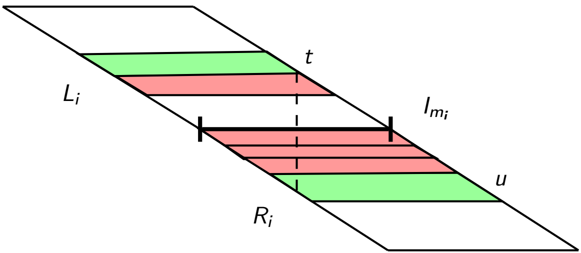
PTAS in Proper Interval Graphs



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Then:

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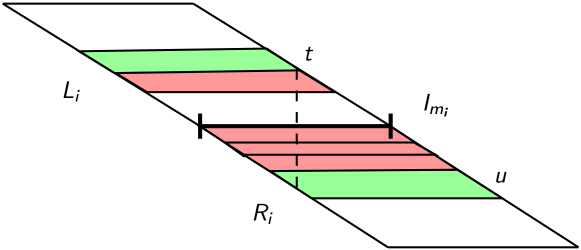
PTAS in Proper Interval Graphs



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Finally, we can prove that $\frac{SOL}{OPT} \leq 1 + \frac{4}{P}$

PTAS in Proper Interval Graphs

Conclusion:

Theorem

For any P, the previous algorithm outputs a $(1 + \frac{4}{P})$-approximation for the k-Sparsest Subgraph in Proper Interval graphs in $O(n^{O(P)})$

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- recursive call with :
  - $k' \leftarrow k' - |T|$
  - $C' \leftarrow C - E(T)$
  - $s \leftarrow$ left endpoint of the rightmost interval after T

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Given a set $\mathcal{I}$ of intervals, $k \leq |\mathcal{I}|$ and a cost C^*

Idea of the algorithm:

- we sort intervals according to their right endpoints
- parameters of the dynamic programming:
  $k' \leftarrow k$, $C' \leftarrow C^*$, $s \leftarrow$ left endpoint of the leftmost interval
- given the parameters, we construct ~~all~~ subsets T s.t.
  (i) T is connected
  (ii) T starts after s (i.e. to the right of s)
  (iii) $|T| \leq k'$ and $E(T) \leq C'$
- recursive call with :
  - $k' \leftarrow k' - |T|$
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  - $s \leftarrow$ left endpoint of the rightmost interval after T
- $\Rightarrow$ at most $k.C^*.n$ different inputs
  what about the running time of one call ?

FPT Algorithm in Interval Graphs

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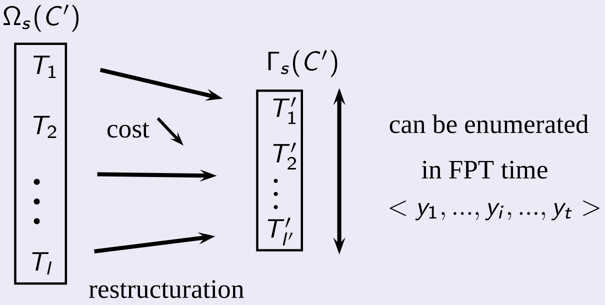

FPT Algorithm in Interval Graphs

Restructuration of a connected component T.

We process intervals of T using a cursor S_i.

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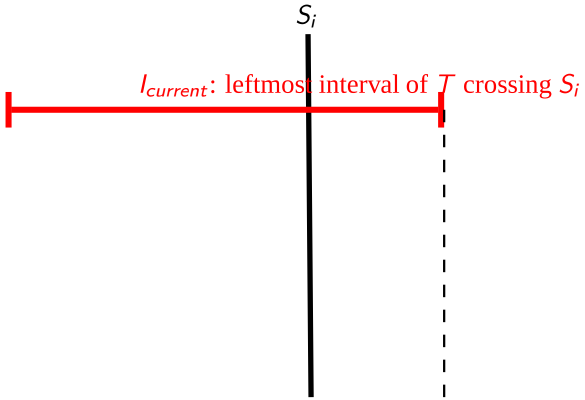

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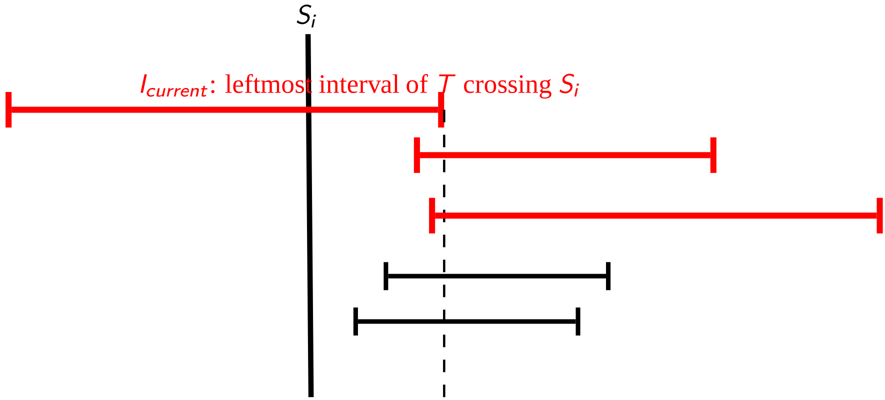

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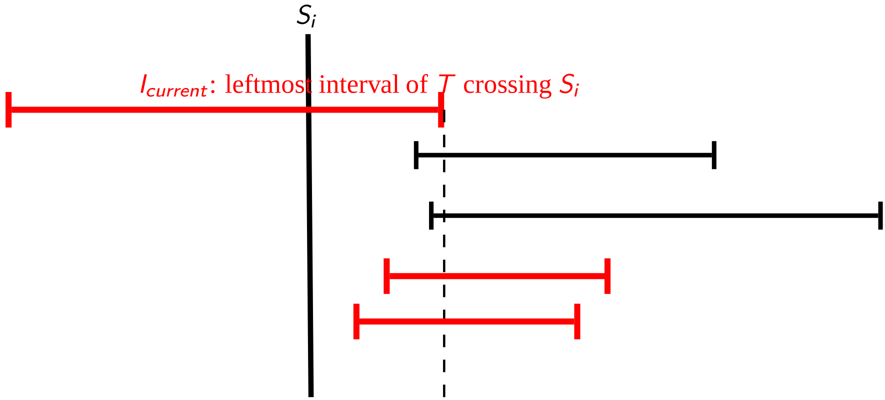

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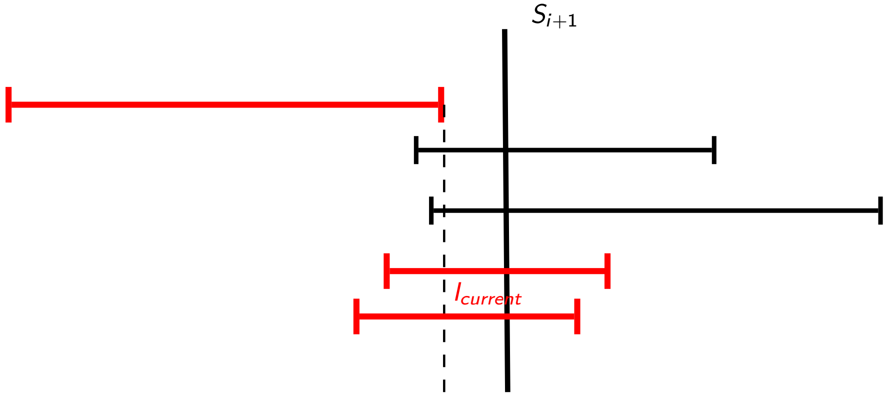

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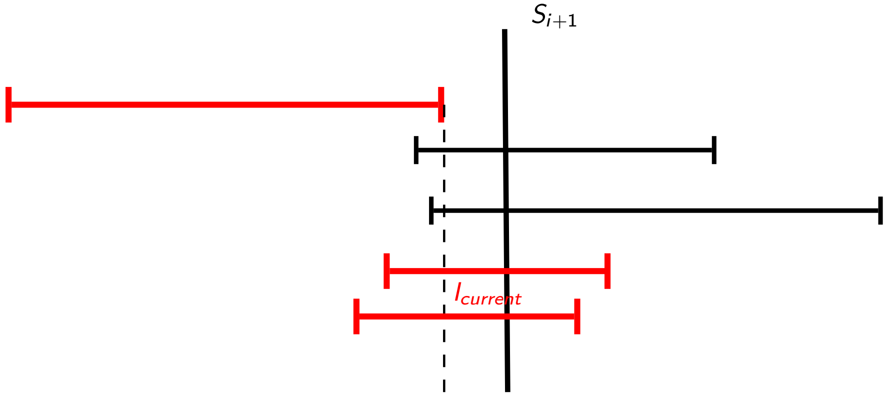

$\Rightarrow$ we only have to "guess" the number y_i of intervals overlapping $I_{current}$

FPT Algorithm in Interval Graphs

Any connected component $T \in \Gamma_s(C)$ can be encoded by a vector $< y_1, ..., y_i, ..., y_t >$

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$$|\Gamma_s(C)| \leq (\sqrt{2C} + 2)^{C+1}$$

and each step of the dynamic programming runs in *FPT* time.

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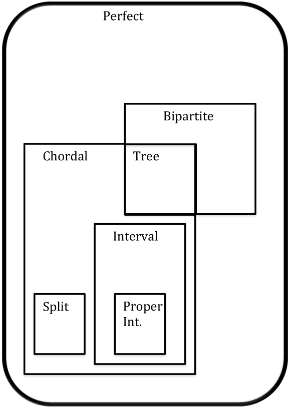
Theorem

Sparsest k-Subgraph in Interval Graphs is *FPT* parameterized by the cost of the solution.

Contents

Open problems and Future Work



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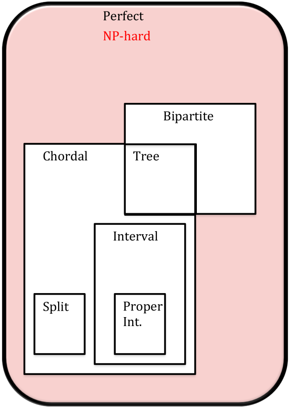

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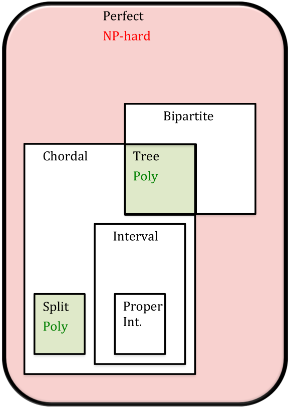

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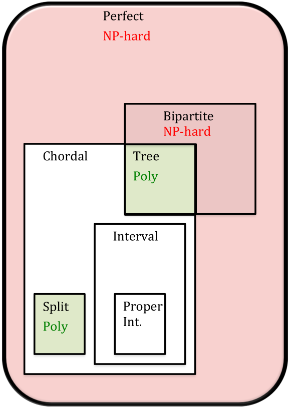

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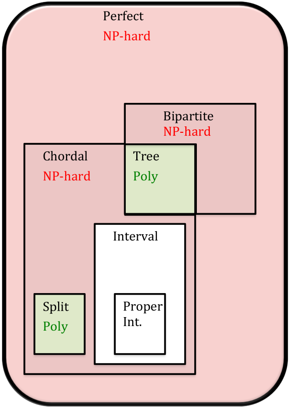

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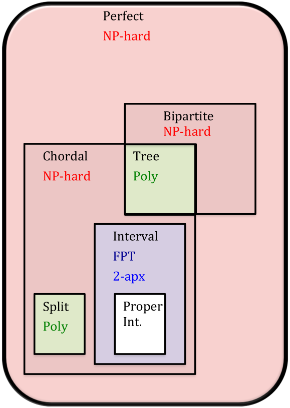

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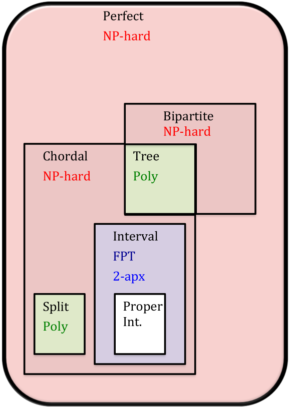

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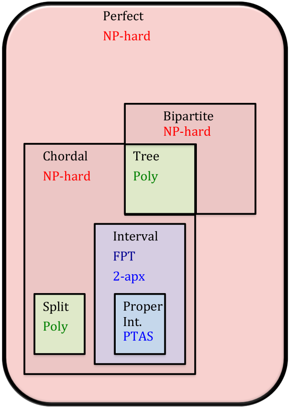

- *NP*-h/Poly in (Proper) Interval graphs ?
- extend FPT and/or approximation results to Chordal graphs ?
- polynomial kernel in Interval graphs ?

Thank you for your attention!