Resiliency problems: algorithms and applications in access control

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(* work mainly done when I was at RHUL)
Joint work with: Jason Crampton, Gregory Gutin and Martin Koutecký

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   - Resiliency, definition of the problem
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Resiliency: general idea

Decision problem

Definition of a problem = set of instances, set of positive instances
Goal: Given an instance $\mathcal{I}$, is $\mathcal{I}$ positive ?

Examples of classical (NP-hard) problems:

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- $\mathcal{I}$ is a CNF formula, positive iff it is satisfiable
- $\mathcal{I}$ is a set of subsets of a universe and an integer k, positive iff there exists k sets whose union is the universe
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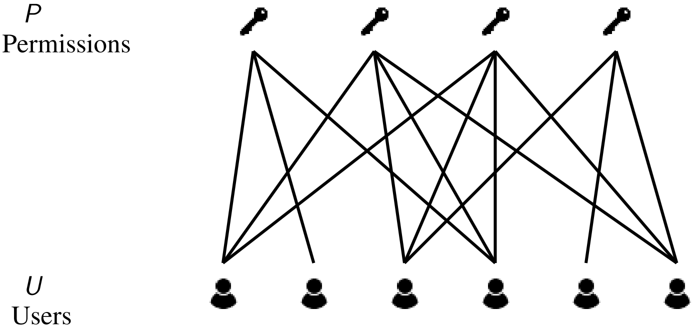
Resiliency problem

Definition of a problem = set of instances, set of positive instances, and for every instance $\mathcal{I}$, a set $Pert(\mathcal{I})$ of perturbed instances
Goal: Given an instance $\mathcal{I}$, is $\mathcal{I}_p$ positive for every $\mathcal{I}_p \in Pert(\mathcal{I})$?

Resiliency Checking Problem (RCP)

Input: an authorization policy: $UP \subseteq U \times P$
$s, d, t \in \mathbb{N}$



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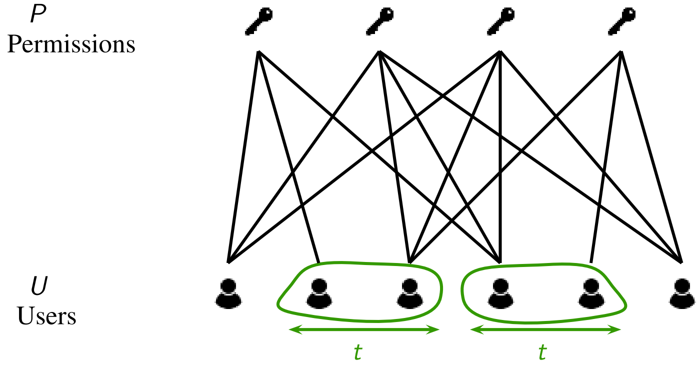

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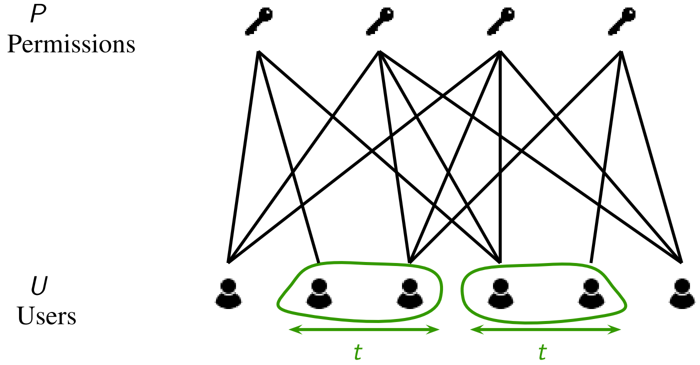

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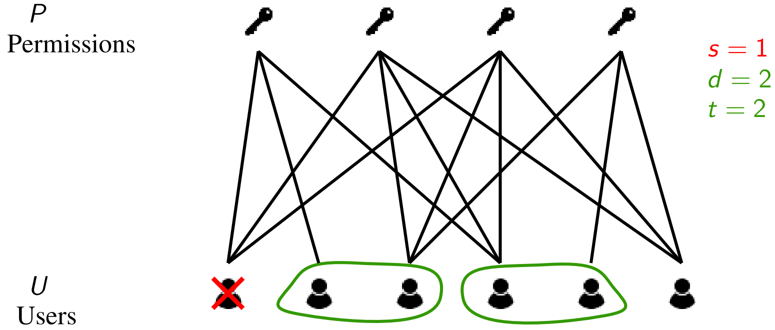

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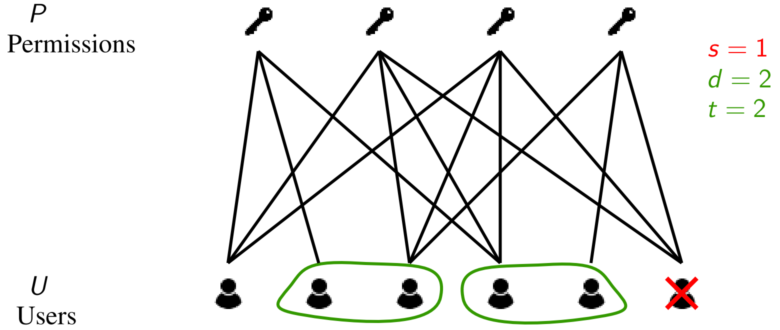

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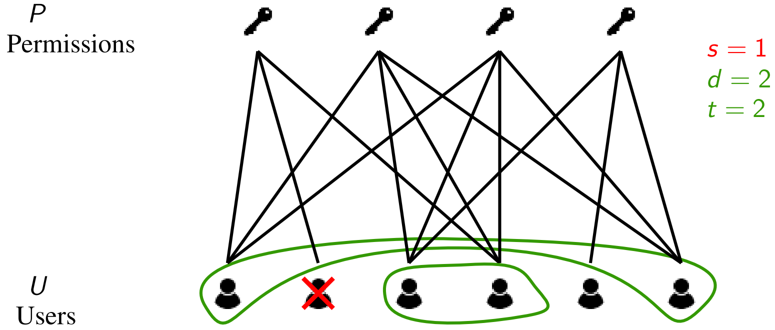

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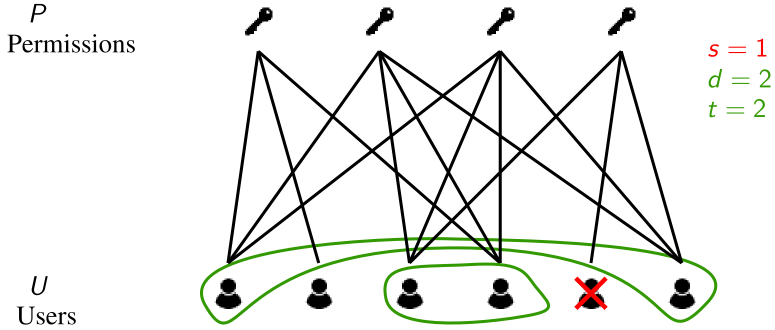

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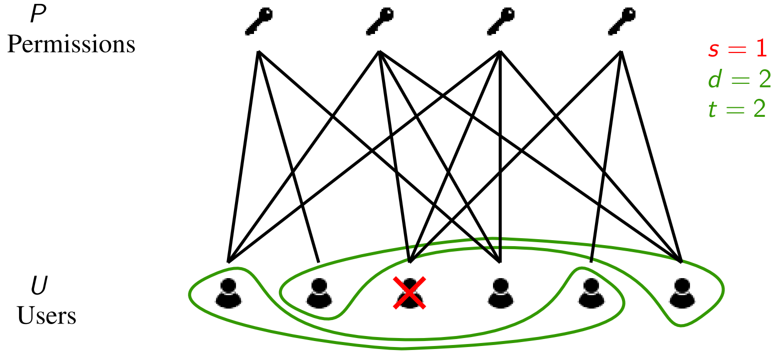

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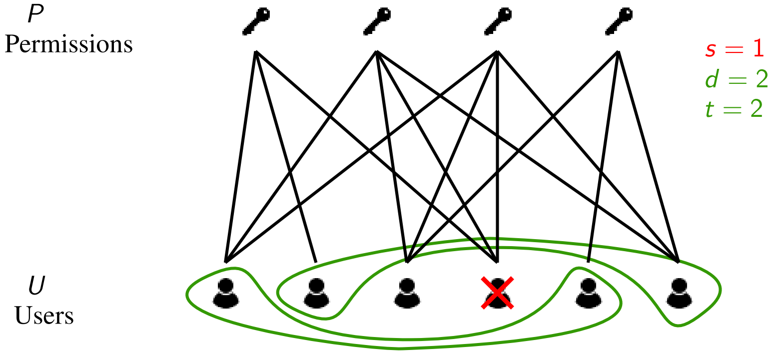

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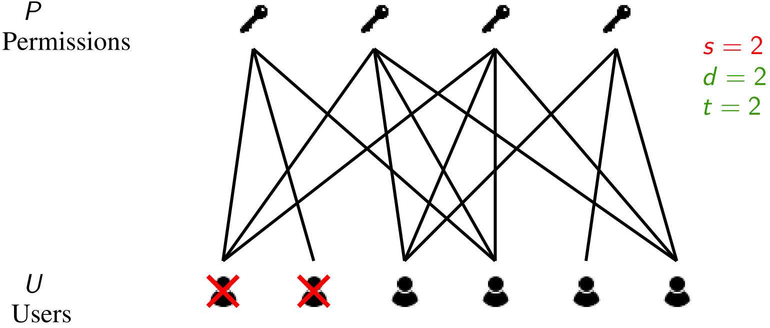

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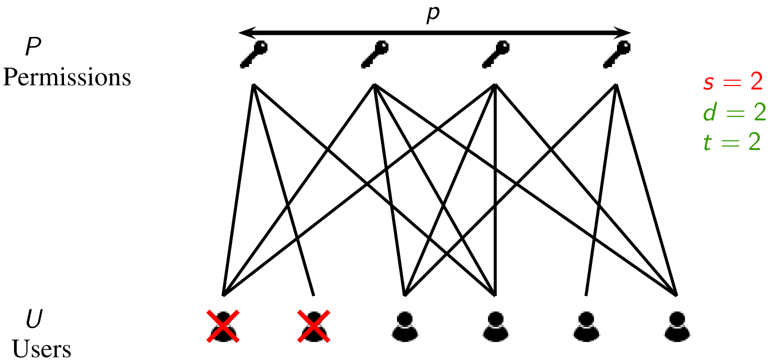

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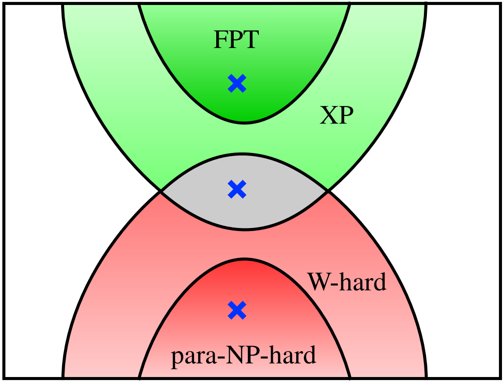

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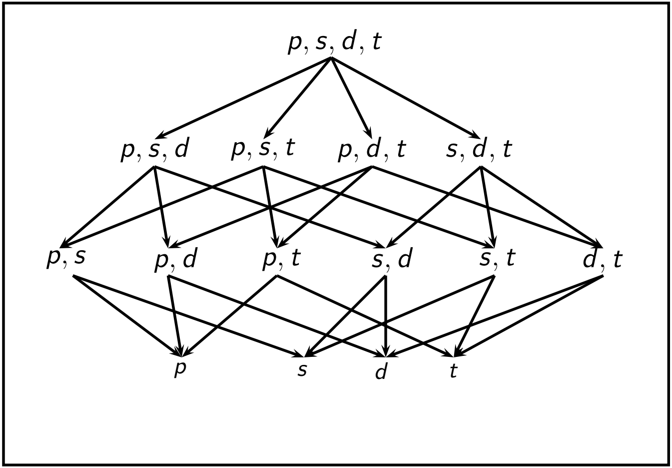

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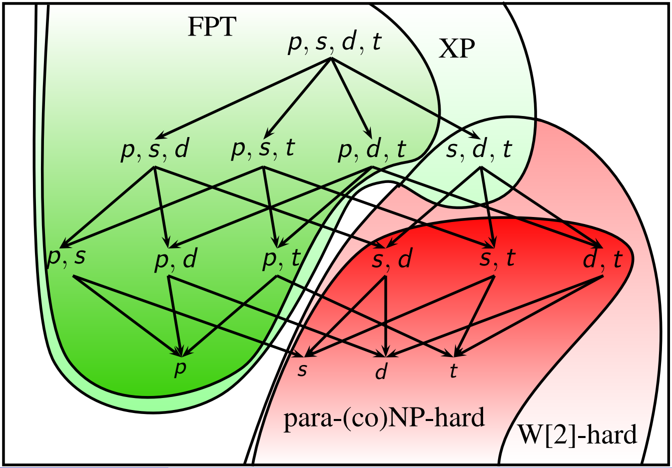

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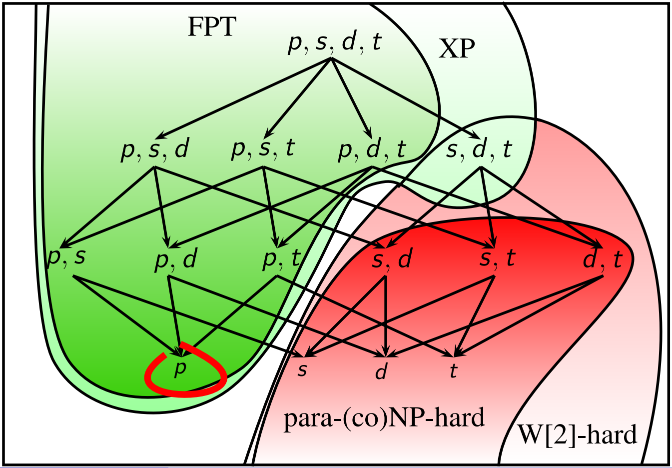

Integer Linear Programs

Example of an ILP:

$$-x_1 + x_2 \leq 1$$

$$3x_1 + 2x_2 \leq 12$$

$$6x_1 - 3x_2 \geq 0$$

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Theorem [Lenstra, 1983]+[Kannan, 1987]+[Frank and Tardos, 1987]

Whether a given Integer Linear Program (ILP) has a non-empty solution set can be decided in $O^*(n^{2.5n+o(n)})$ time and polynomial space, where n is the number of variables.

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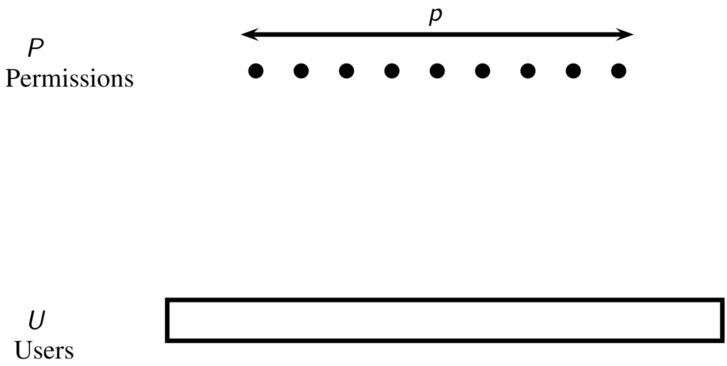

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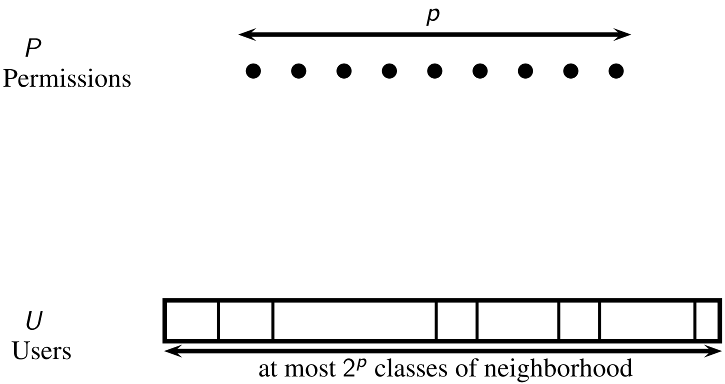
- partition U into at most 2^p groups of users of same neighborhood



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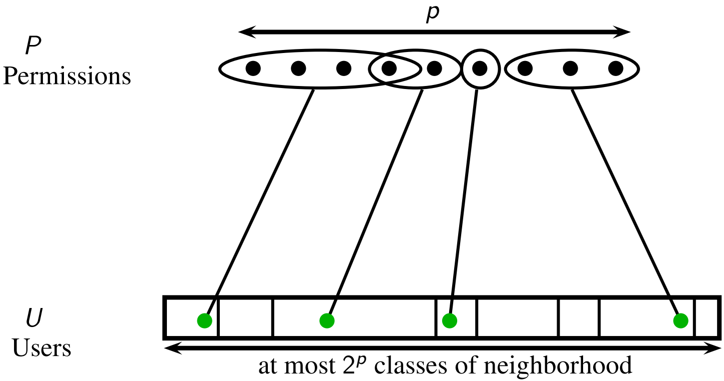

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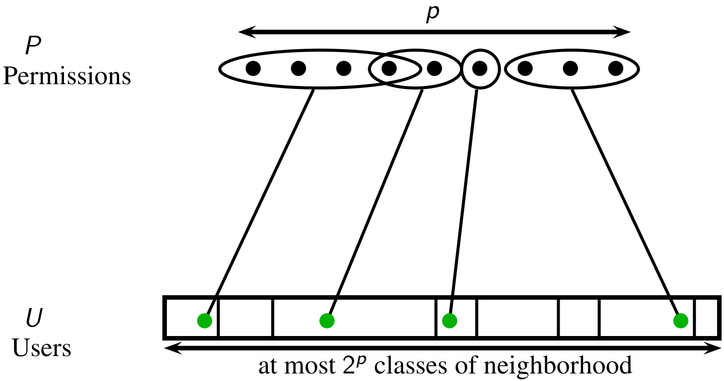

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- Second constraint:

$$\sum_{c \in \mathcal{C}[N]} x_c \leq |U[N]| \quad \forall N \subseteq P$$

  where:
  - $\mathcal{C}[N]$ are the configurations involving N
  - $U[N]$ = users having neighborhood N

From RCP<$s = 0$> to RCP<>

To find a set of teams: solve the following ILP:

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What we need to solve: for every assignment of variables z_N, is there an assignment of variables x_c ?

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- F_X: conjunction of inequalities involving variables X only
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Now: how to solve it in FPT time parameterized by the number of variables, constraints, and unary size of encoding of matrices of the ILPs.

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Idea of the algorithm:

- eliminate variables X and obtain an equivalent disjunction of ~~ILPs~~

$$L_1 \vee \cdots \vee L_r$$

- then: test whether there exists an assignment of variables Z such that

$$\neg L_1 \wedge \cdots \wedge \neg L_r$$

  is satisfied

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Then: replace all inequalities of L and G, by:

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Solution [Williams, 1076]:

$$\bigvee_{h \in \{0,\dots,a-1\}} \begin{array}{c} b^g - \sum_{k=2}^{n} a_k^g x_k + h \leq b^\ell - \sum_{k=2}^{n} a_k^\ell x_k \\ \text{and} \\ b^g - \sum_{k=2}^{n} a_k^g x_k + h \equiv 0 \mod a \end{array} \quad \forall \ell \in L, \forall g \in G \tag{9}$$

$\rightarrow$ Back to ILP resiliency

Z-resiliency: for any assignment of Z satisfying F_Z, does there exist an assignment of X such that both assignments satify $F_X \wedge F_Y \wedge F_{XY}$?

Idea of the algorithm:

- eliminate variables X and obtain an equivalent disjunction of systems of linear inequalities and congruences (SLIC)

$$L_1 \vee \cdots \vee L_r$$

- then: test whether there exists an assignment of variables Z such that

$$\neg L_1 \wedge \cdots \wedge \neg L_r$$

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(such a s is called a d-closest string)

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Result: Resiliency Closest String is FPT parameterized by k.

Generalization to other domains

- Closest String
- Scheduling: Makespan Minimization on Unrelated Machines
- Computational social choice: Swap Bribery
- ...?

Conclusion

- ILP-resiliency provides a very general framework
  Other applications?
- The known algorithm is FPT parameterized by:
  - number of variables
  - number of constraints
  - encoding length of the matrices of the ILPs

  $\Rightarrow$ can we do better?
  - using less parameters?
  - or: prove a lower bound: ILP-resiliency W[.]-hard parameterized by number of variables (and constraints) only ?

Voilà !
Questions ?