Multidimensional Binary Vector Assignment problem: standard, structural and above guarantee parameterizations

Rémi Watrigant[1]

joint work with Marin Bougeret[2], Guillerme Duvillié[2], Rodolphe Giroudeau[2]

[1] Hong Kong Polytechnic University, Hong Kong
[2] LIRMM, Montpellier, France

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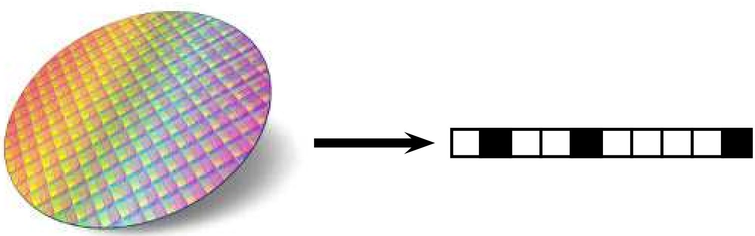
Contents

Applications

Yield maximization in wafer-to-wafer 3D chip integration.

Multidimensional Binary Vector Assignment

- a wafer = a binary vector of good/bad dies (1/0)



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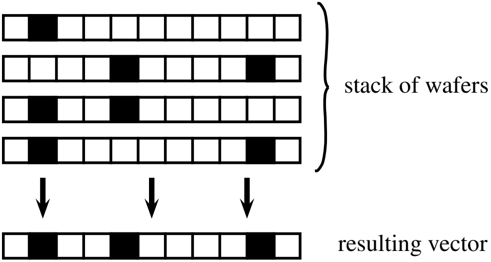

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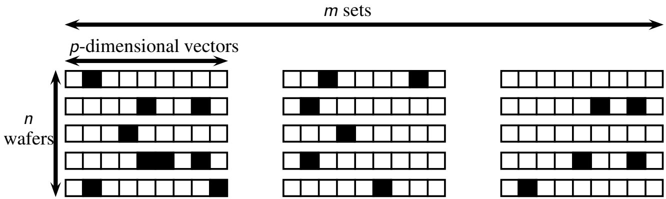

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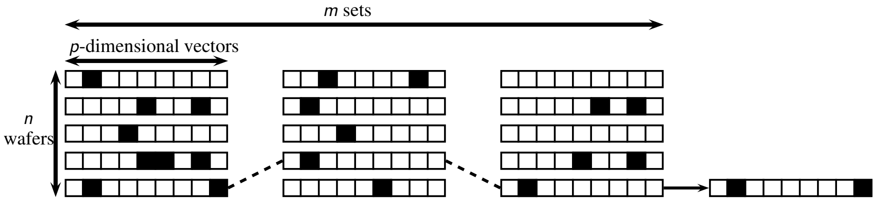

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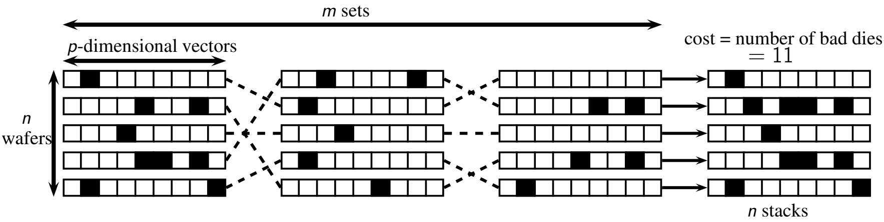

Related works

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  - $W[1]$-hard for standard parameter (maximization version)

Parameterized algorithms

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Fixed-Parameter Tractability

A problem is **FPT** if there is an algorithm solving any instance $\mathcal{I}$ in time $O(\ f(\kappa(\mathcal{I}))\ \ poly(|\mathcal{I}|)\ )$

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- Exponential Time Hypothesis: we suppose that 3-SAT cannot be solved in time $O^*(2^{o(n)})$ (n = number of variables)

First observations

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- Input: m sets of n wafers (p-dimensional binary vectors)
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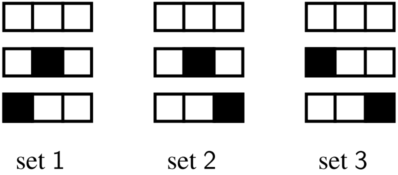
Warmup: is it alway safe to create "full-good stacks" ?



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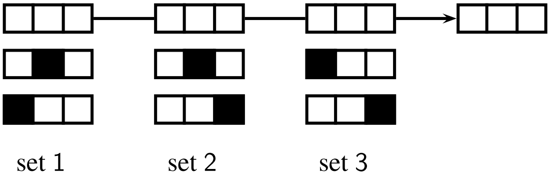

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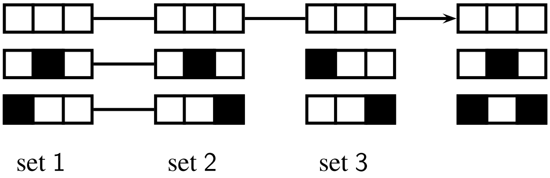

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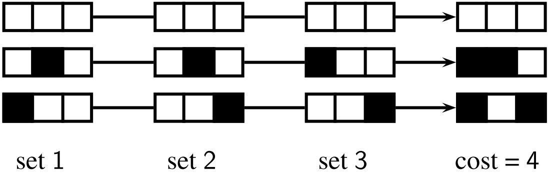

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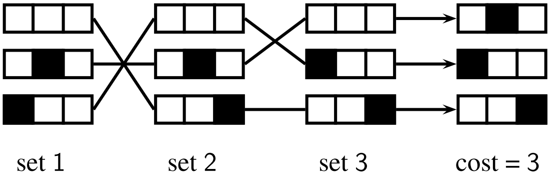
Warmup: is it alway safe to create "full-good stacks" ? No!



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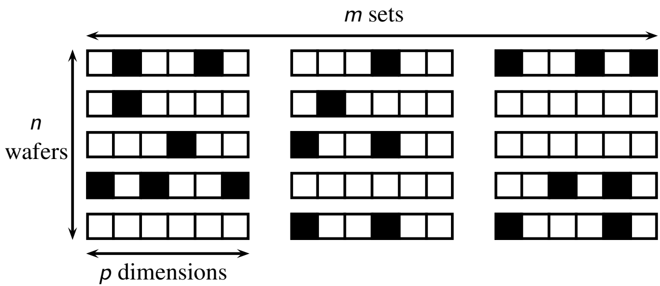

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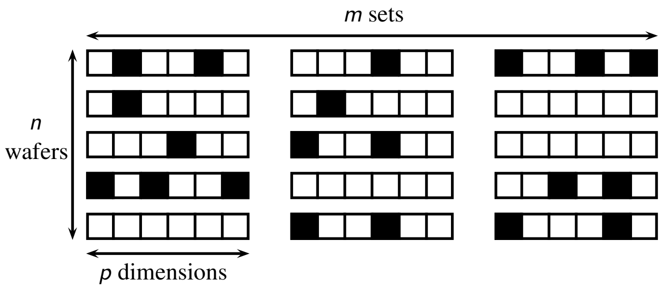

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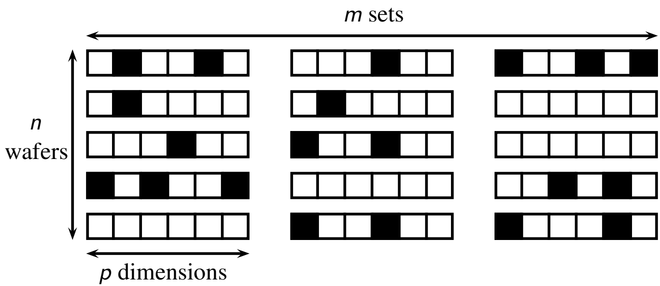

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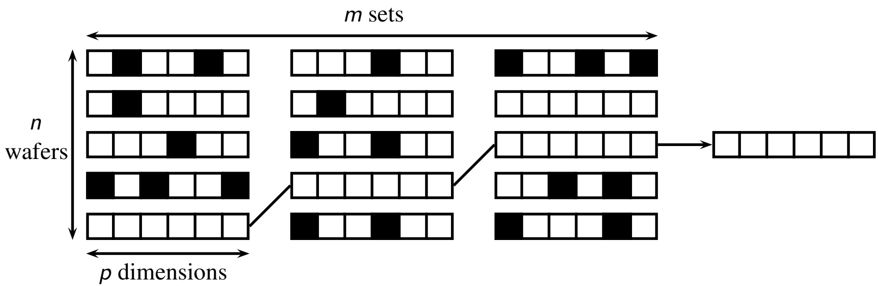

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$\Rightarrow$ we can arbitrarily form a full-good stack

remove it, and continue until $n \leq k$

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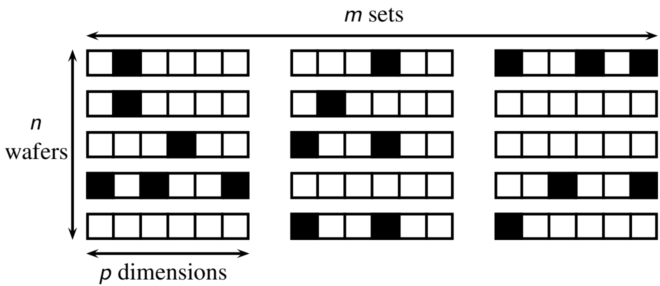

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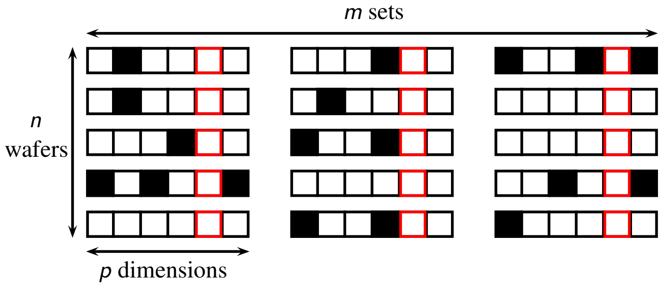

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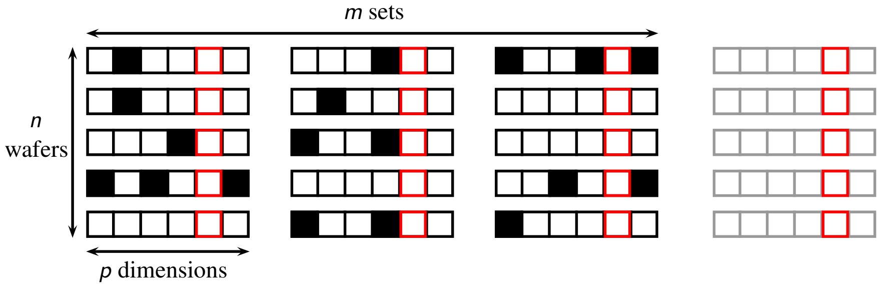

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$\Rightarrow$ any solution will also have this good die

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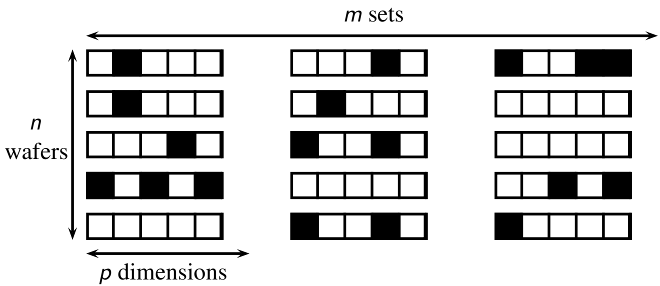

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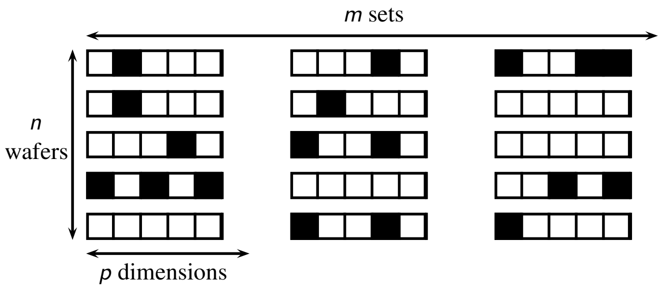

- if there is a component with good dies everywhere

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- we can remove the component from the instance

$\Rightarrow$ the cost of any solution will be at least p

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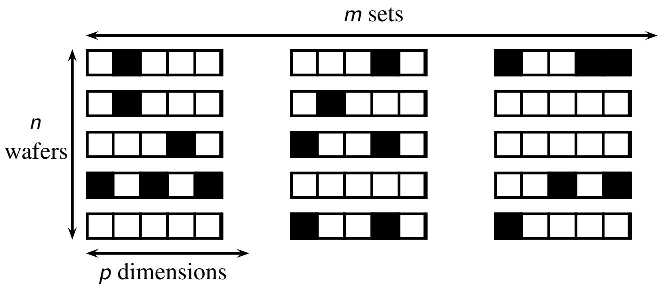

Theorem

- From any instance, we can produce in polynomial time an equivalent one of size at most $O(k^2 m)$ $\Rightarrow$ kernel of size $O(k^2 m)$

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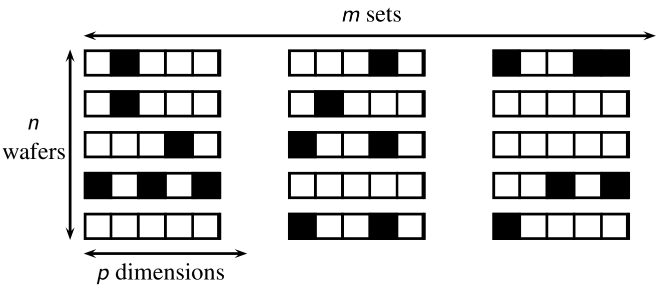

Theorem

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- No kernel polynomial in p even for $m = 3$ (unless $NP \subseteq coNP/poly$)

Above guarantee parameterization

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Idea: substracting a lower bound to the objective function

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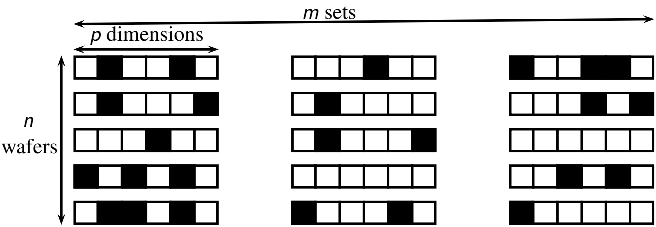

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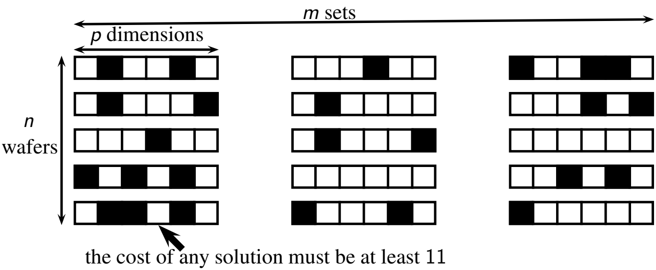

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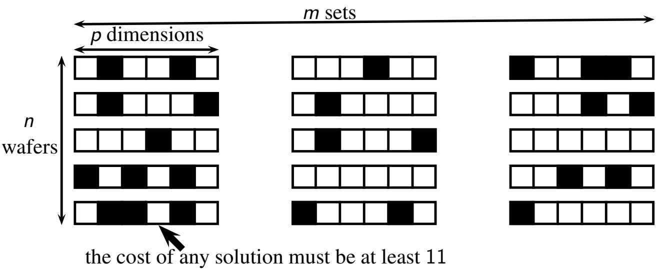
Idea: substracting a lower bound to the objective function
Let $\mathcal{B}$ be the maximum number of bad dies in a set
$\Rightarrow$ parameter $\zeta_{\mathcal{B}} = k - \mathcal{B}$



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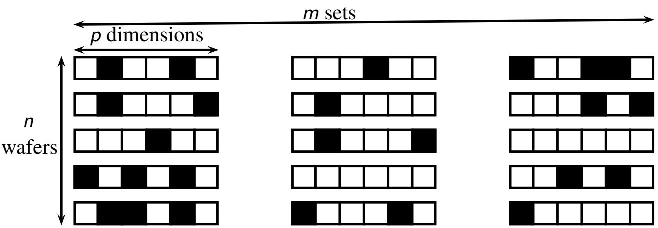
- Find in polynomial time if there is a costless assignment between two sets



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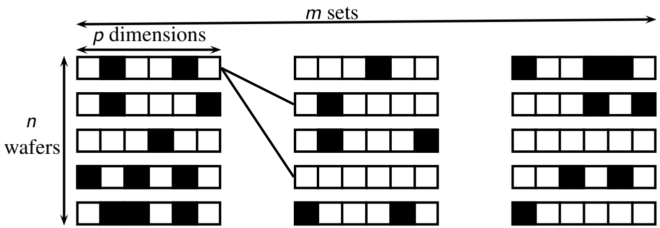

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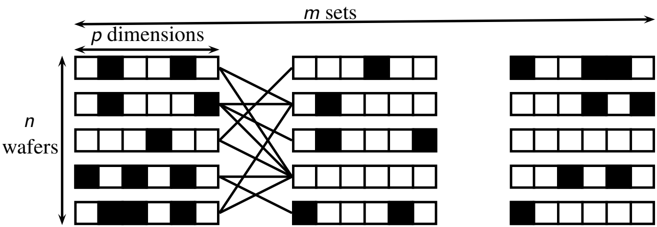

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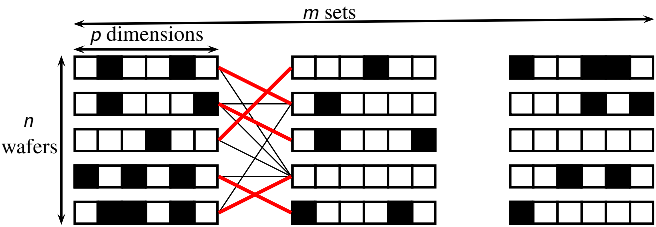

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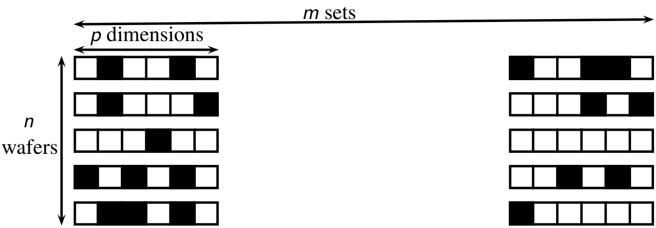

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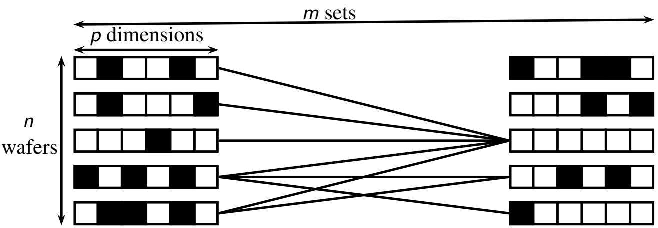

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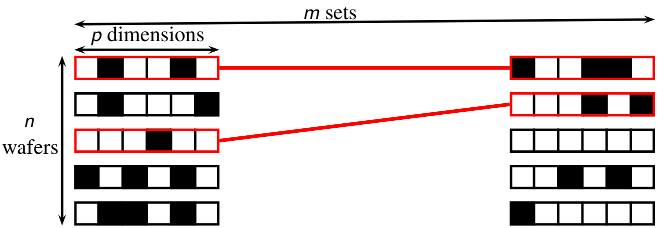

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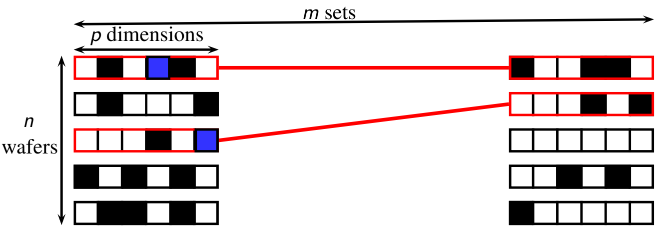

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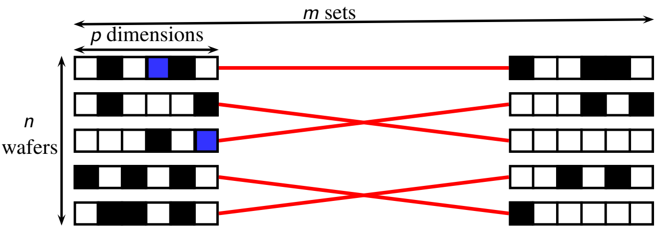

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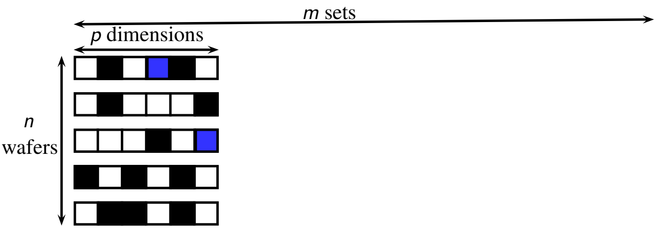

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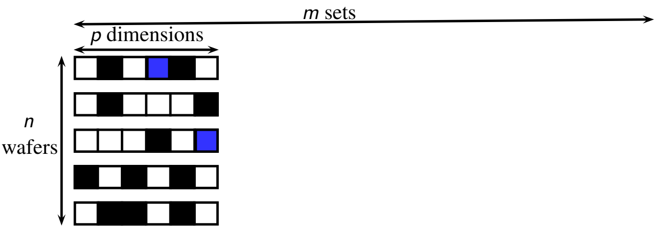

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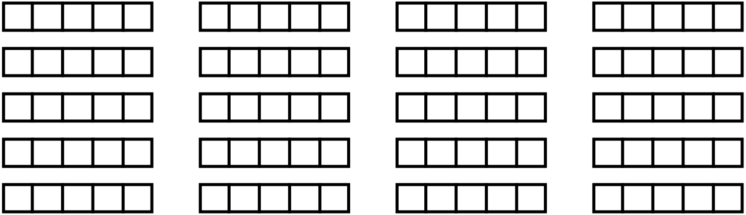

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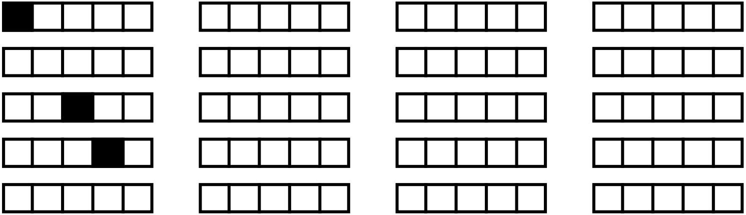

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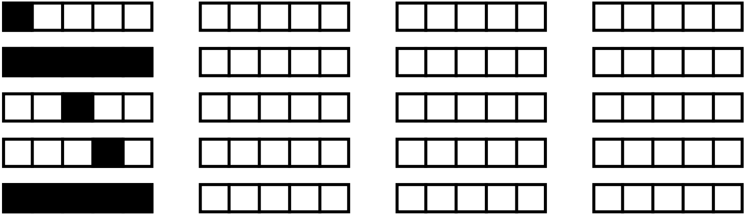

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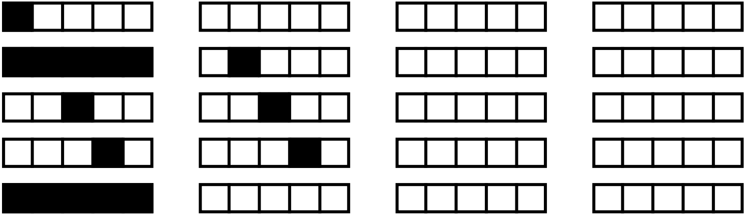

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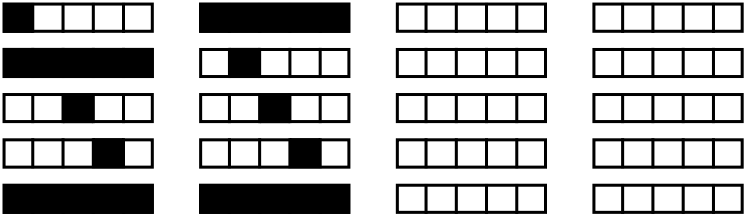

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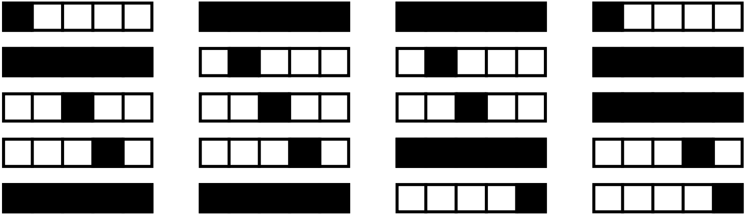

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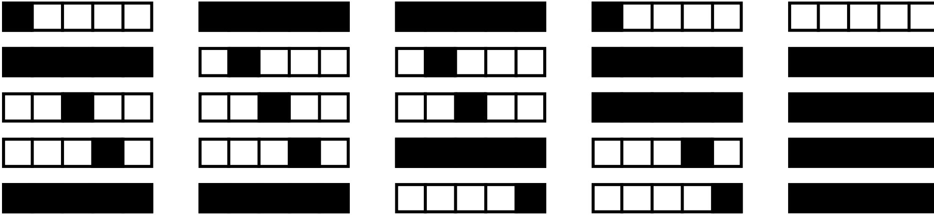

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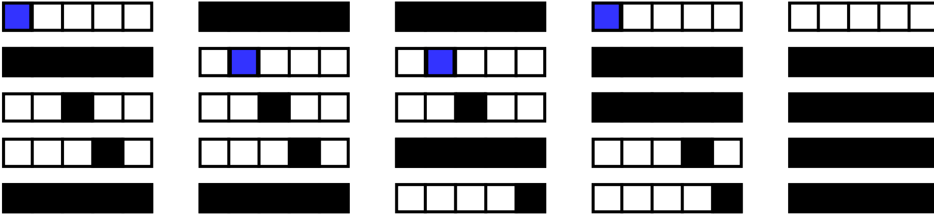

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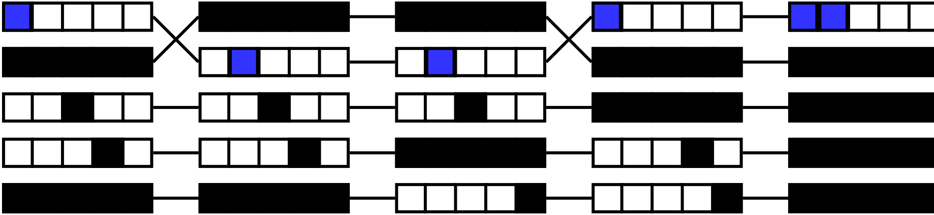

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$\Rightarrow$ no $O^*(2^{o(\zeta_{\mathcal{B}}) \log(n)})$ nor $O^*(2^{\zeta_{\mathcal{B}} o(\log(n))})$ for our problem, under *ETH*.

Summary of the results

Positive results	Negative results
$O(k^2 m)$ kernel	no $p^{O(1)}$ kernel unless $NP \subseteq coNP/poly$
$O^*(4^{\zeta_{\mathcal{B}} \log(n)})$ algorithm	$W[2]$-hard for $\zeta_{\mathcal{B}}$ only no $2^{o(\zeta_{\mathcal{B}}) \log(n)}$ nor $2^{\zeta_{\mathcal{B}} o(\log(n))}$ under ETH no $2^{o(k)}$ for fixed n under ETH
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Open questions:

- algorithm in $O^*(2^k)$? (n part of the input)
- polynomial kernel parameterized by k only ?

Thank you for your attention !