Cardinality constrained subgraph problems

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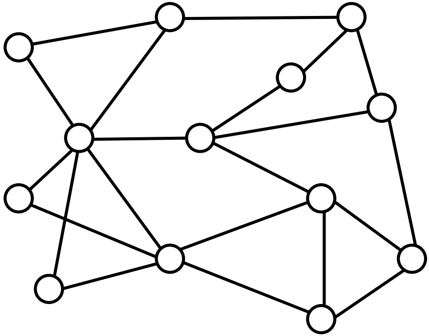
results picked from two works:
one with É. Bonnet, M. Bougeret, V. Paschos, F. Sikora
one with M. Bougeret, N. Bousquet, R. Giroudeau

Workshop on Parameterized Complexity
Chofu, Tokyo, Japan
Feb 28 - Mar 1, 2015

Introduction

Cardinality constrained subgraph problem:
Input: a graph $G = (V, E)$, $k \in \mathbb{N}$
Output: a set S of k vertices



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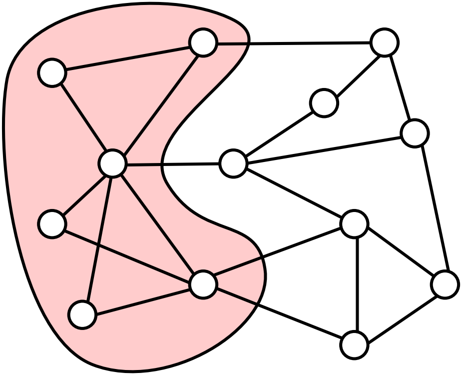
$k = 6$



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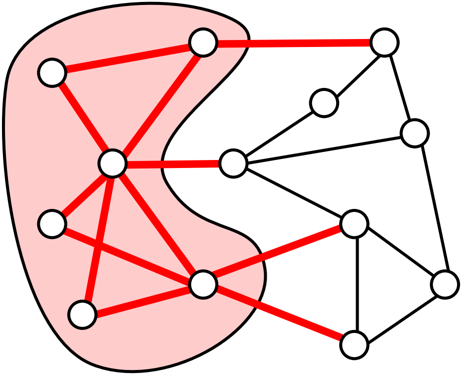

Maximum k-Cover
Minimum k-Cover

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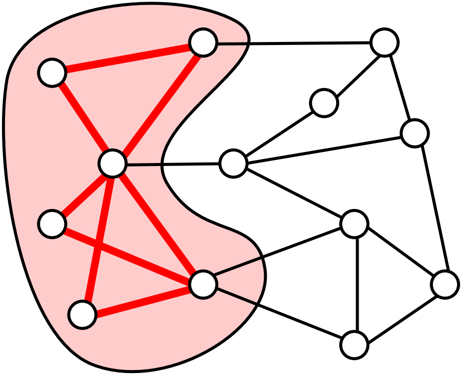

Densest k-Subgraph
Sparsest k-Subgraph

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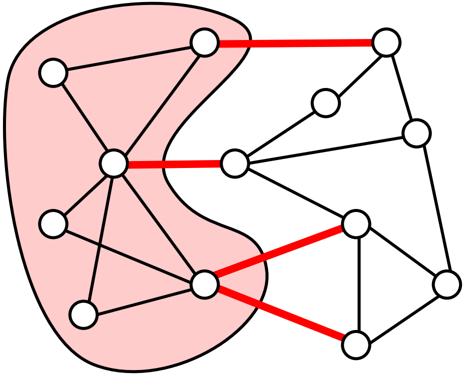

Maximum $(k, n - k)$-Cut
Minimum $(k, n - k)$-Cut

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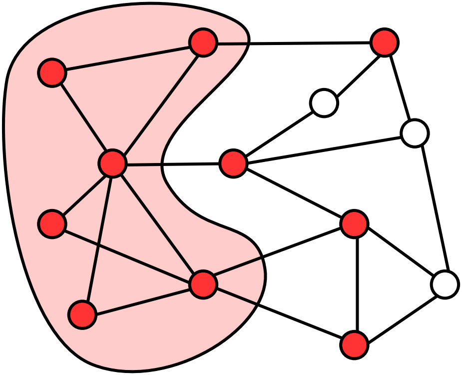
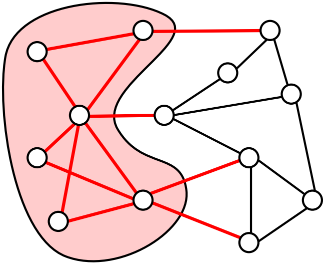
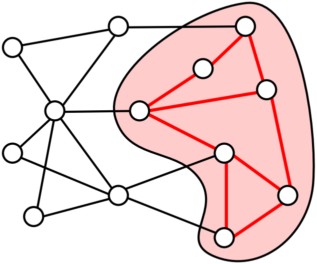


Maximum k-Dominating Set
Minimum k-Dominating Set

Introduction





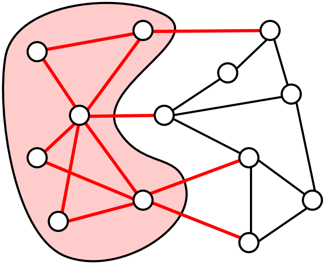
Maximum k-Cover

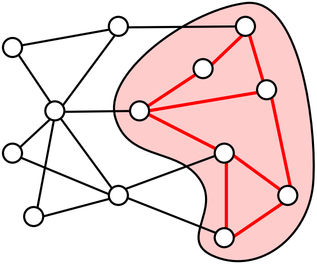
Densest k-Subgraph

Minimum k-Cover

Sparsest k-Subgraph

Introduction





Maximum k-Cover

complement

$G \leftrightarrow \bar{G}$

Minimum k-Cover

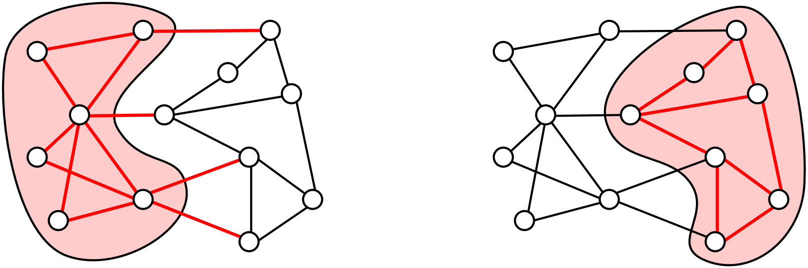
Densest k-Subgraph

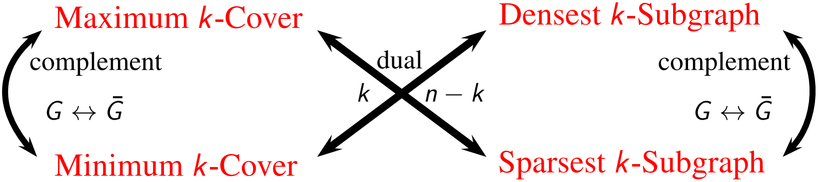
complement

$G \leftrightarrow \bar{G}$

Sparsest k-Subgraph

Introduction





Known results/summary

Graphs	Covering		Inducing	
	Max Max-k-Cover	Min Min-k-Cover	Max Densest	Min Sparsest
General				
Bipartite				
Chordal				

Known results/summary

Graphs	Covering		Inducing	
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General	NP-h (generalization of Clique, Independent Set)			
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General	NP-h (generalization of Clique, Independent Set) $W[1]$-h (k) [Cai '08]			
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Graphs	Covering		Inducing	
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General	NP-h (generalization of Clique, Independent Set) $W[1]$-h (k) [Cai '08] FPT(std. param.) [Blaser'03]			
Bipartite				
Chordal				

Known results/summary

	Covering		Inducing	
Graphs	Max	Min	Max	Min
	Max-k-Cover	Min-k-Cover	Densest	Sparsest
General	*NP*-h (generalization of Clique, Independent Set)			
	$W[1]$-h (k) [Cai '08]			
	FPT (std. param.) [Blaser'03]			
Bipartite	*NP*-h [CK'14]			
Chordal				

Known results/summary

Graphs	Covering		Inducing	
	Max Max-k-Cover	Min Min-k-Cover	Max Densest	Min Sparsest
General	*NP*-h (generalization of Clique, Independent Set) $W[1]$-h (k) [Cai '08]			
	FPT (std. param.) [Blaser'03]			
Bipartite	*NP*-h [CK'14]		*NP*-h [CP'84] $W[1]$-h [CP'84]	
Chordal				

Known results/summary

Graphs	Covering: Max (Max-k-Cover)	Covering: Min (Min-k-Cover)	Inducing: Max (Densest)	Inducing: Min (Sparsest)
General	*NP*-h (generalization of Clique, Independent Set) $W[1]$-h (k) [Cai '08] *FPT* (std. param.) [Blaser'03]			
Bipartite	*NP*-h [CK'14]	*NP*-h (dual)	*NP*-h [CP'84] $W[1]$-h [CP'84]	*NP*-h (dual)
Chordal				

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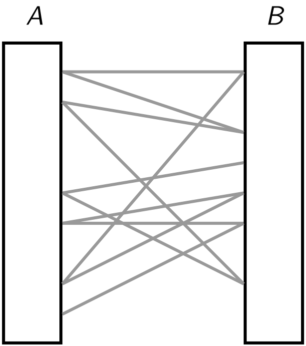
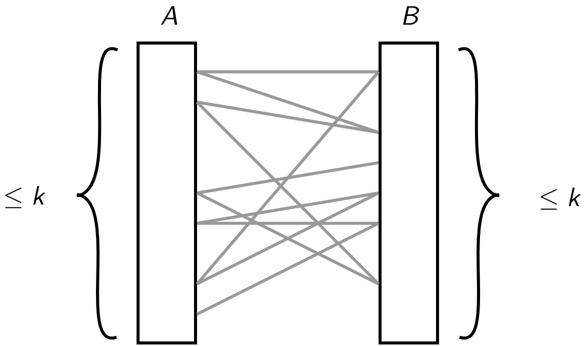
Bipartite	*NP*-h [CK'14]	*NP*-h (dual)	*NP*-h [CP'84] $W[1]$-h [CP'84]	*NP*-h (dual) *FPT*(k)[trivial]
Chordal				


A
B



A
B
$\leq k$
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Known results/summary

Graphs	Covering		Inducing	
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Bipartite	*NP*-h [CK'14] *FPT*(k) [BBPSW'14]	*NP*-h (dual) $W[1]$-h [BBPSW'14]	*NP*-h [CP'84] $W[1]$-h [CP'84]	*NP*-h (dual) *FPT*(k)[trivial]
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Bipartite	*NP*-h [CK'14] *FPT*(k) [BBPSW'14]	*NP*-h (dual) $W[1]$-h [BBPSW'14]	*NP*-h [CP'84] $W[1]$-h [CP'84]	*NP*-h (dual) *FPT*(k)[trivial]
Chordal		*NP*-h (dual)	*NP*-h [CP'84] *FPT*(k) [trivial]	

Densest k-Subgraph in chordal graphs

- if G has a clique of size k (polynomial)

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  $\Rightarrow$ optimal solution

- otherwise:

Densest k-Subgraph in chordal graphs

- if G has a clique of size k (polynomial)
  $\Rightarrow$ optimal solution
- otherwise: treewidth $\leq k$
  $\Rightarrow$ classical dynamic programming gives $O^*(2^k)$ algorithm

Known results/summary

Graphs	Covering: Max Max-k-Cover	Covering: Min Min-k-Cover	Inducing: Max Densest	Inducing: Min Sparsest
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Chordal		*NP*-h (dual)	*NP*-h [CP'84] *FPT*(k) [trivial]	

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Graphs	Covering: Max Max-k-Cover	Covering: Min Min-k-Cover	Inducing: Max Densest	Inducing: Min Sparsest
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Chordal	*NP*-h (dual)	*NP*-h (dual)	*NP*-h [CP'84] *FPT*(k) [trivial]	*NP*-h [BBGW'14] *FPT*(k) [BBBGW'14]

FPT algorithm for Max k-Cover in bipartite graphs

Input: $G = (V, E)$, $k \in \mathbb{N}$, a set $V' \subseteq V$ in which we have to pick the solution

Lemma 1

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Let $S \subseteq V$ be of size $k-1$, and $u, v \notin S$.
If $d(u) \geq d(v) + k$ then $S \cup \{u\}$ is strictly better than $S \cup \{v\}$

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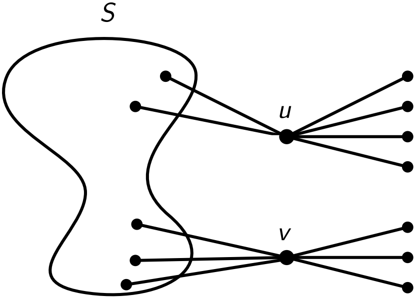

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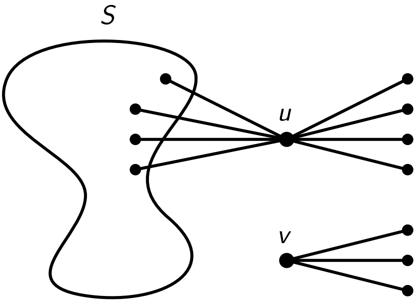

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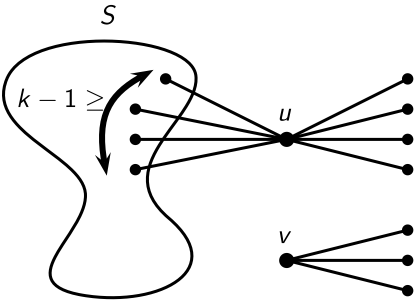

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For all $v \in V'$, $d(v) \in [\Delta - 2k, \Delta]$ (Δ = max. degree of $G[V']$)

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$$\begin{array}{l} d(v_1) = \Delta \\ d(v_2) \\ \vdots \\ d(v_i) \\ \vdots \\ d(v_k) \\ \vdots \\ d(v_j) \\ \vdots \\ d(v_n) \end{array}$$

- sort vertices by non-increasing degrees

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  Proof: suppose $v_j \in opt$
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FPT algorithm for Max *k*-Cover in bipartite graphs

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  Proof: suppose $v_j \in opt$
  then $v_i \notin opt$ for some $i \in \{1, \cdots, k\}$
  but $d(v_i) \geq d(v_k) \geq d(v_j) + k$
  $\Rightarrow$ v_i is better than v_j !

FPT algorithm for Max k-Cover in bipartite graphs

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  then $v_j \in opt$ for some $j > k$
  but $d(v_j) \leq d(v_k) \leq d(v_1) - k$
  $\Rightarrow$ v_1 is better than v_j !

FPT algorithm for Max k-Cover in bipartite graphs

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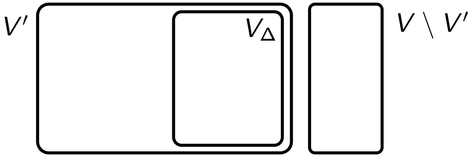

FPT algorithm for Max k-Cover in bipartite graphs

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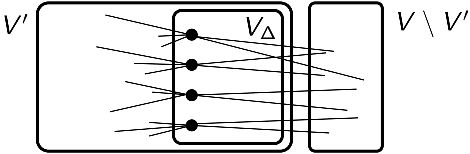

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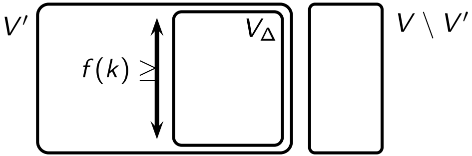

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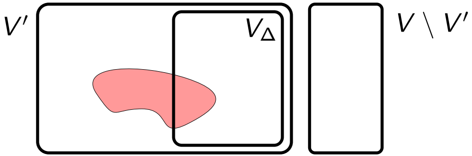

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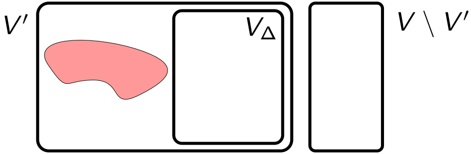

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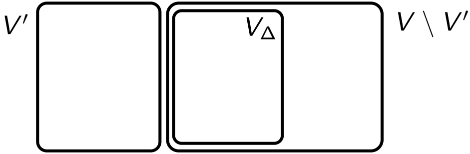

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A similar algorithm can be designed for Max-$(k, n-k)$-Cut

Summary

Graphs	Covering		Inducing	
	Max Max-k-Cover	Min Min-k-Cover	Max Densest	Min Sparsest
General	*NP*-h (generalization of Clique, Independent Set) *W*[1]-h (k) [Cai '08] *FPT*(std. param.) [Blaser'03]			
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Min-k-Cover is $W[1]$-hard in bipartite graphs

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Let $G = (V, E)$, $k \in \mathbb{N}$, and Δ =degree of G

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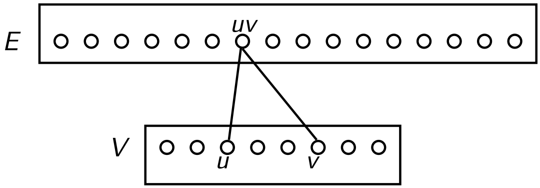

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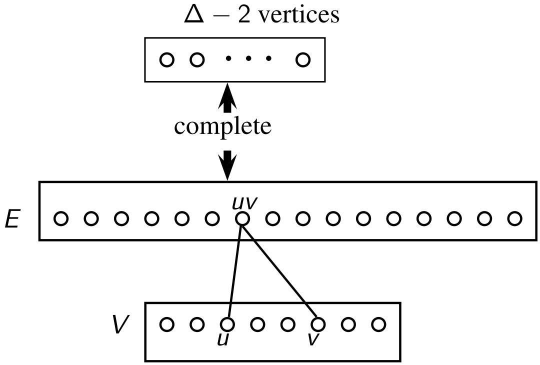

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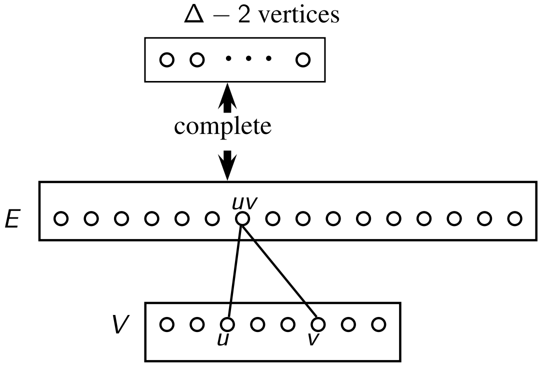

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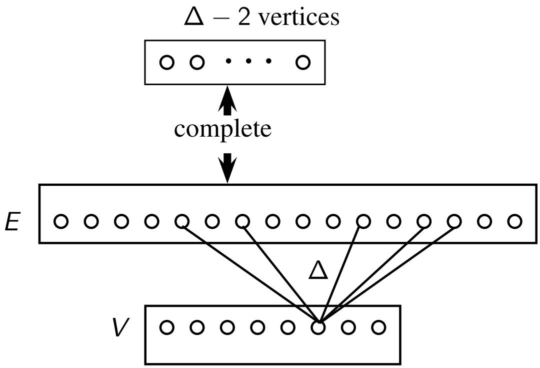

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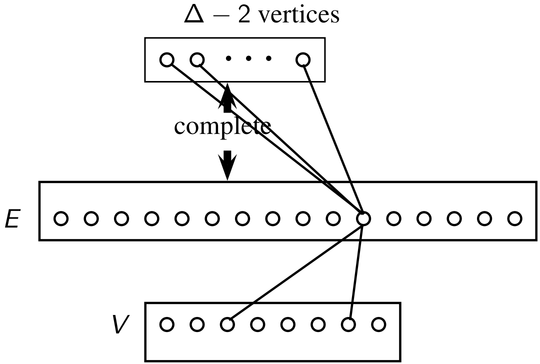

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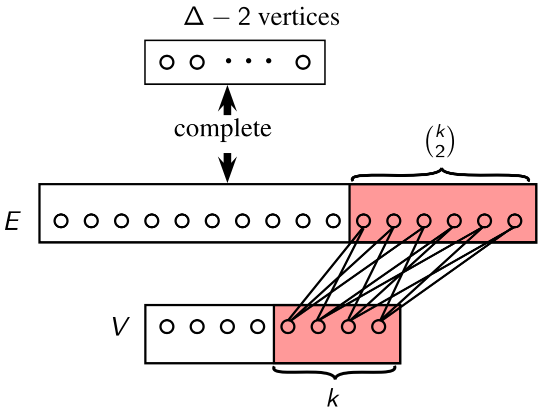

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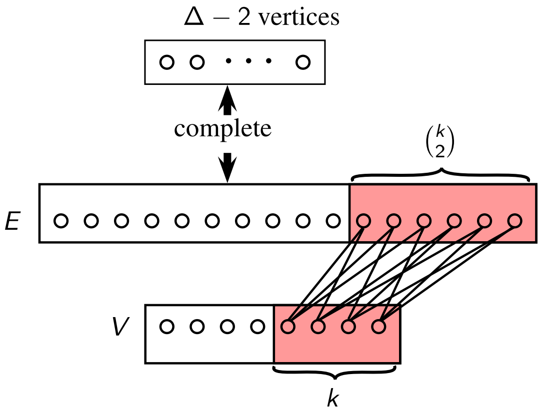
$\Leftrightarrow$ equivalent to find a k'-densest subgraph
$\Leftrightarrow$ equivalent to find a k-clique in G



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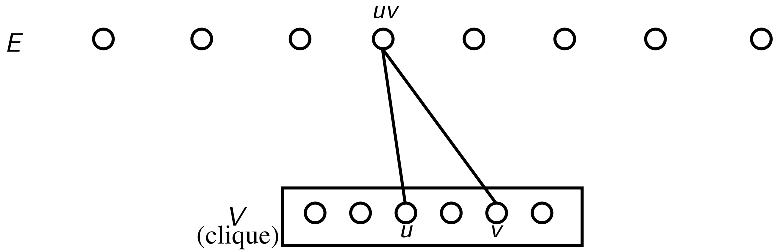

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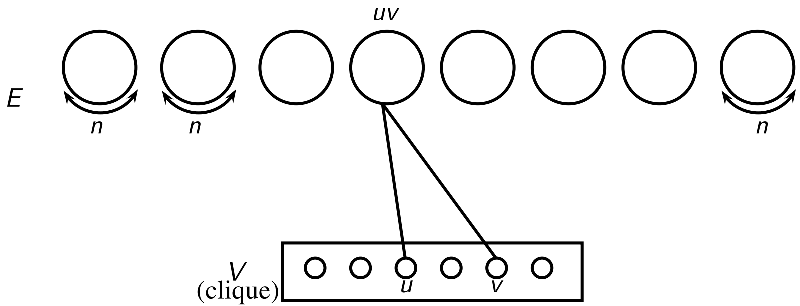

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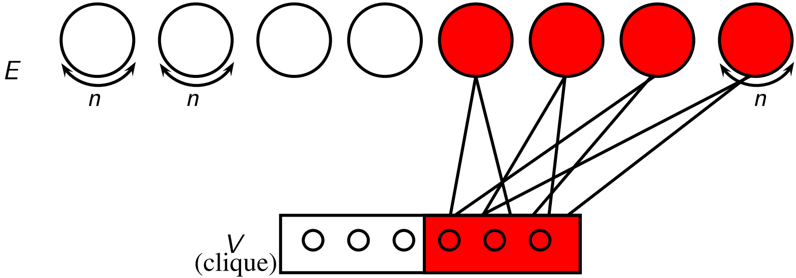

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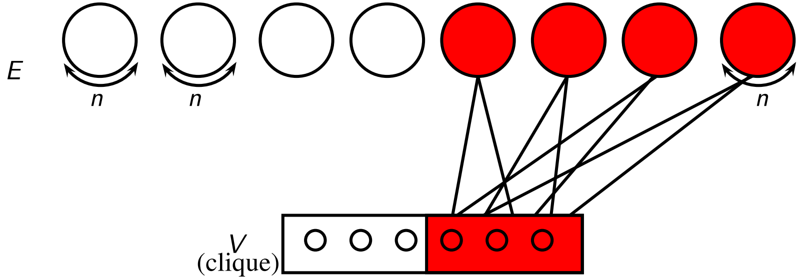

What about Sparsest k-Subgraph now..?

Sparsest *k*-Subgraph is *NP*-hard in chordal graphs [B. G. W., '14]

Idea: gadget

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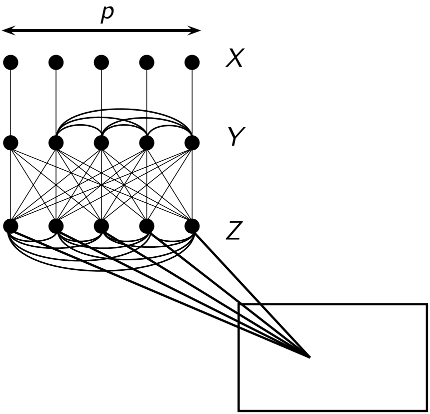

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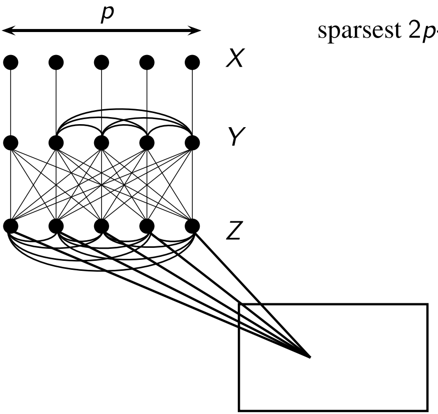

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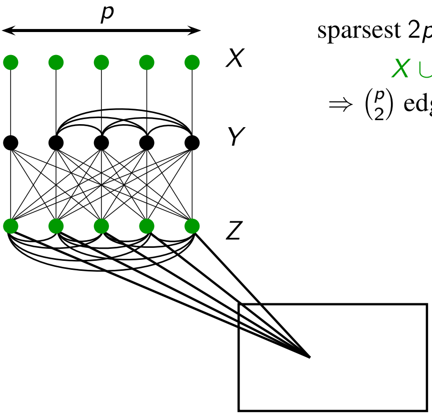

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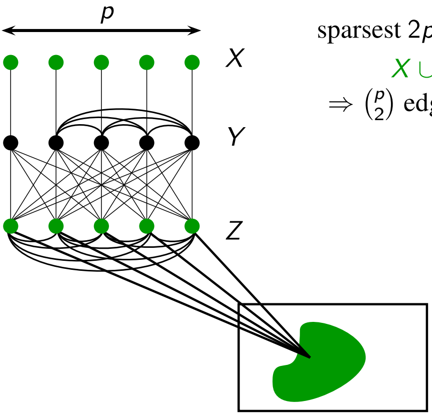

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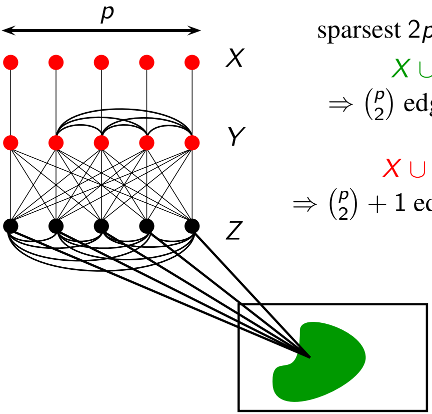

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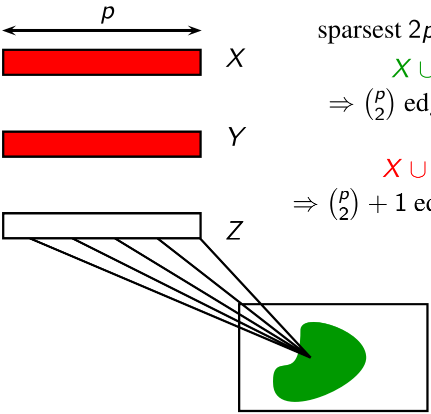

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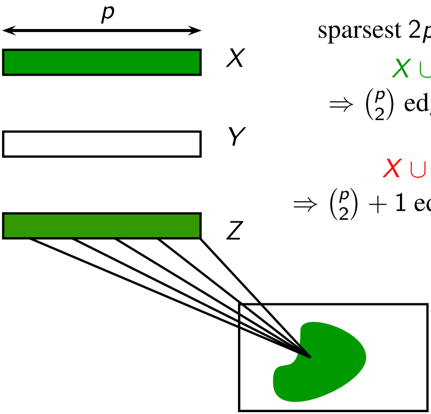

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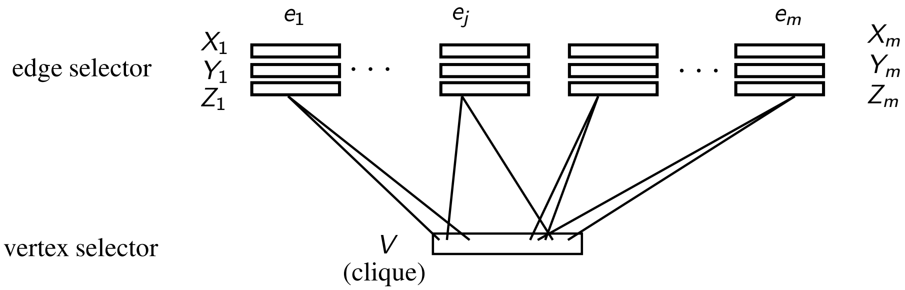

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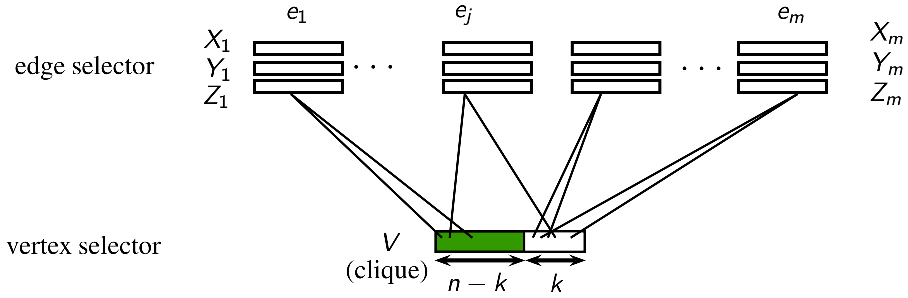

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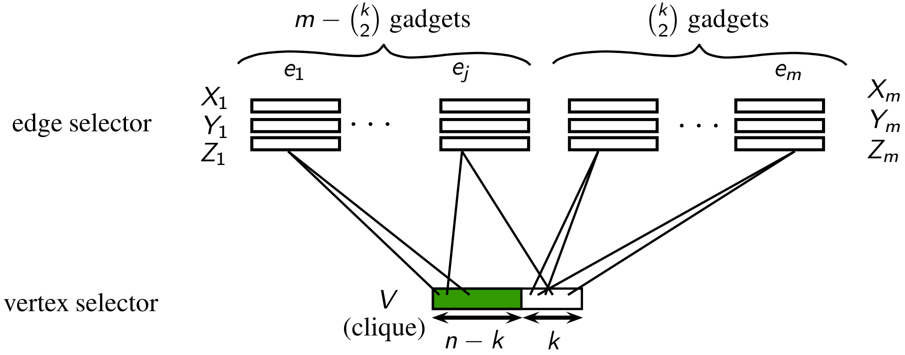

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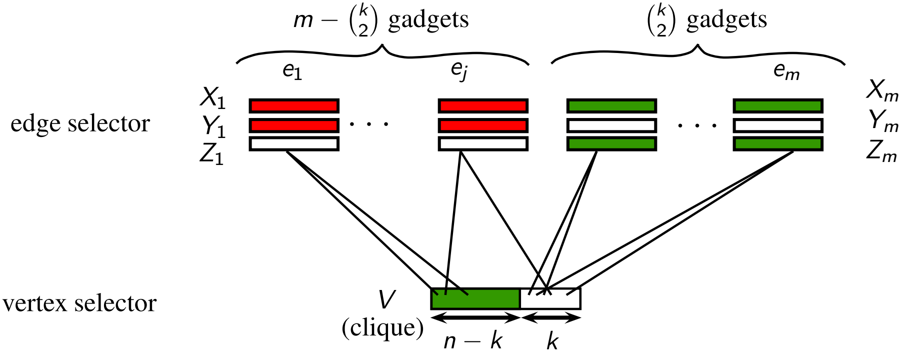

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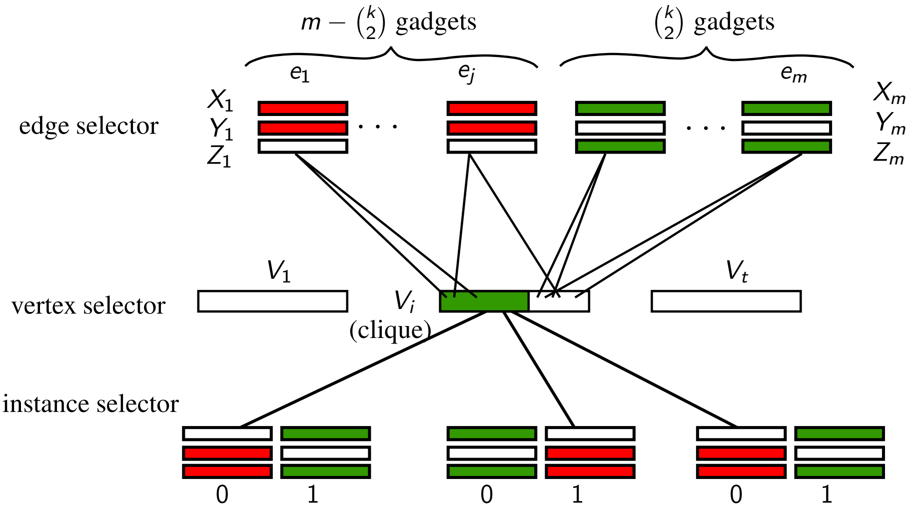
Extension to cross-composition $\Rightarrow$ no polynomial kernel (unless...)



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Sparsest k-Subgraph is FPT in chordal graphs [BBBGW'14]

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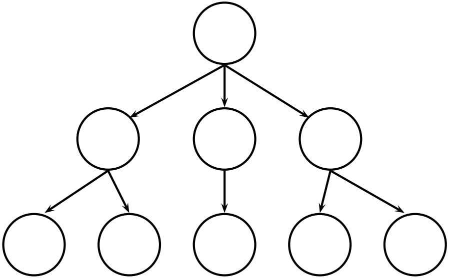

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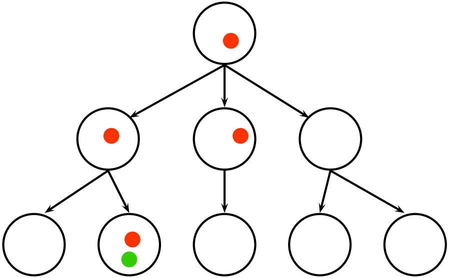

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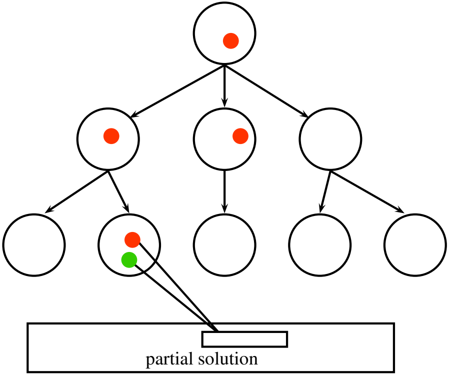

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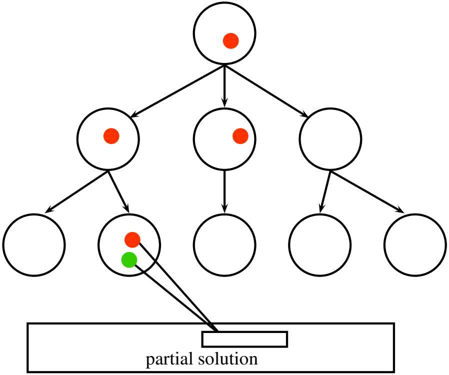

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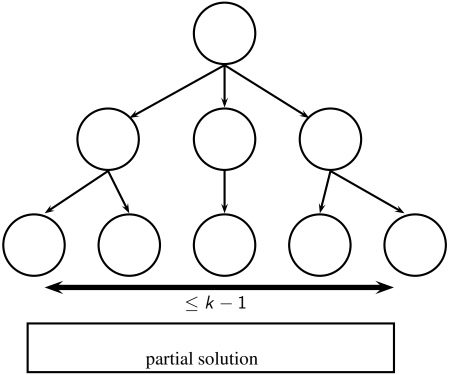

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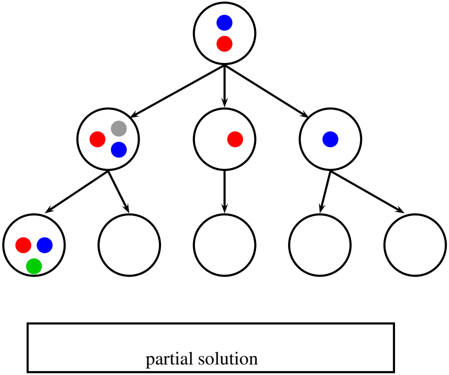

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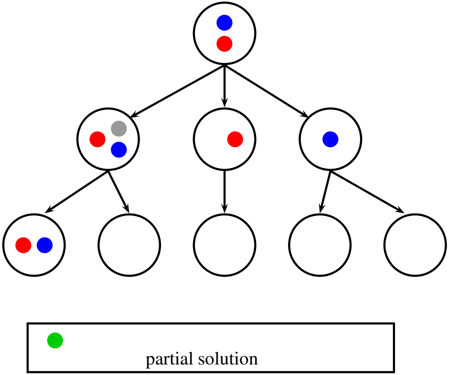

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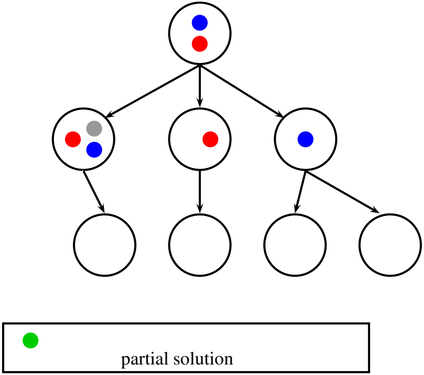

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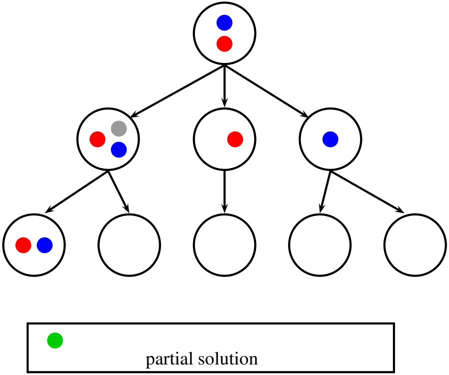

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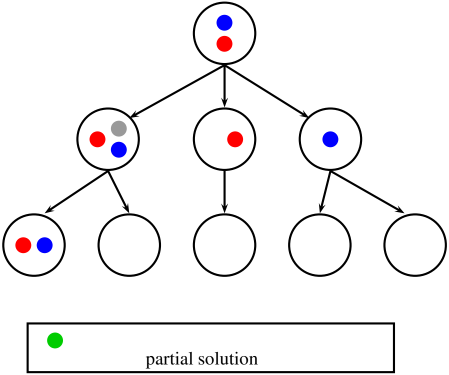

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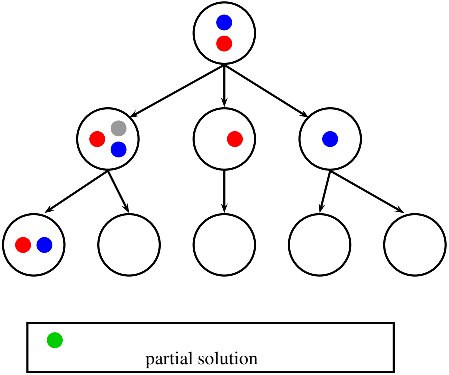

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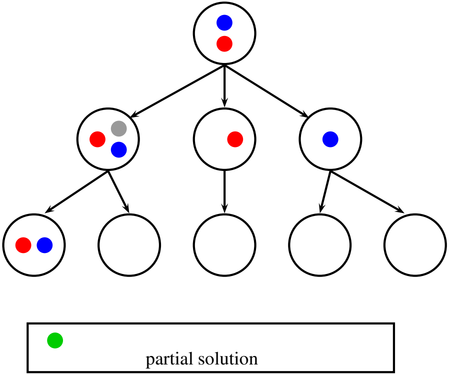

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- but can apply on predecessors of leaves (here: take ●)

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Conclusion, open problems

Graphs	Covering		Inducing	
	Max	Min	Max	Min
	Max-k-Cover	Min-k-Cover	Densest	Sparsest
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(Proper) Interval	?	?	?	?
Planar	*NP*-h (dual)	?	?	*NP*-h (indep. set)

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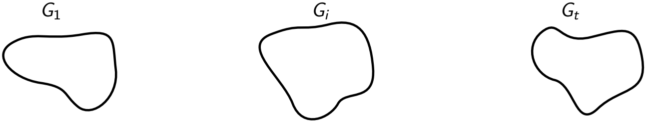

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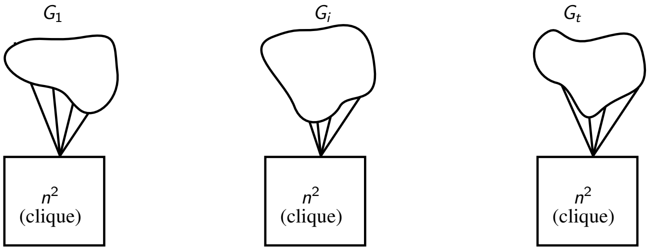

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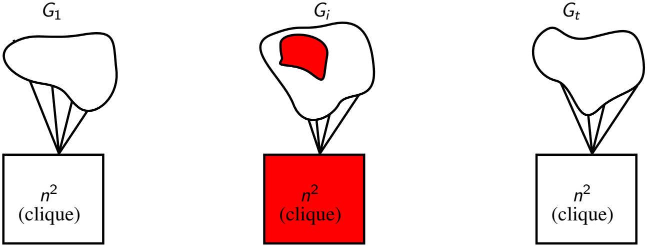

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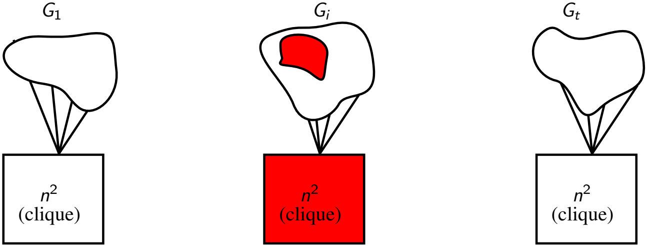

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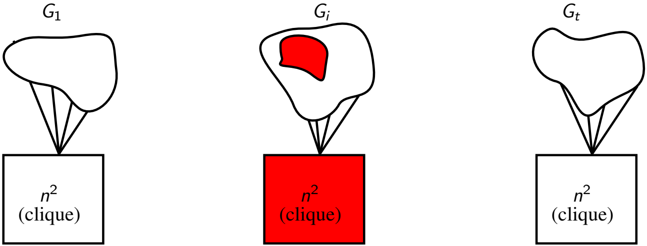

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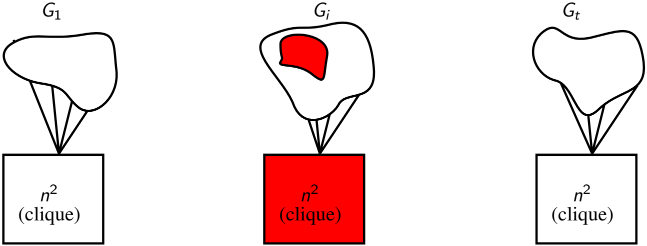

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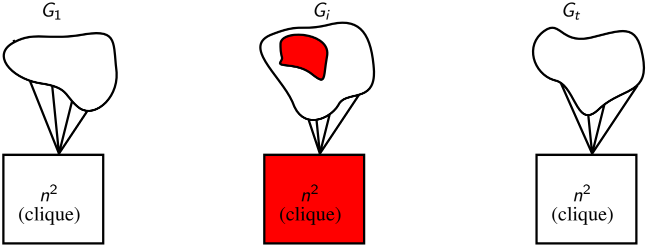

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Merci !