

# On a Parameterized Problem in Access Control

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Joint work with: Jason Crampton and Gregory Gutin

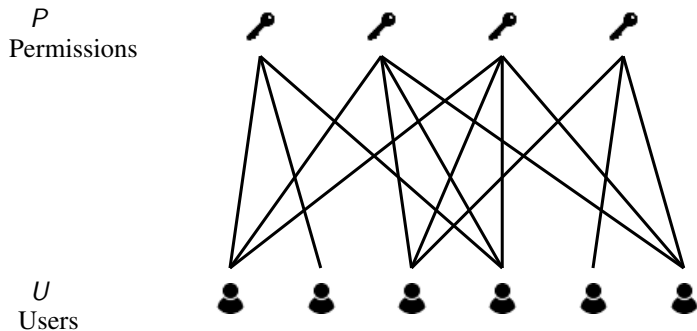
Séminaire COATI - Sophia Antipolis  
March 15th, 2016.

- 1 Definition of the problem
- 2 Appetizer: easy observations
- 3 FPT algorithms
- 4 Efficient algorithms and implementation
- 5 Conclusion

# Resiliency Checking Problem (RCP)

Input: an authorization policy:  $UP \subseteq U \times P$

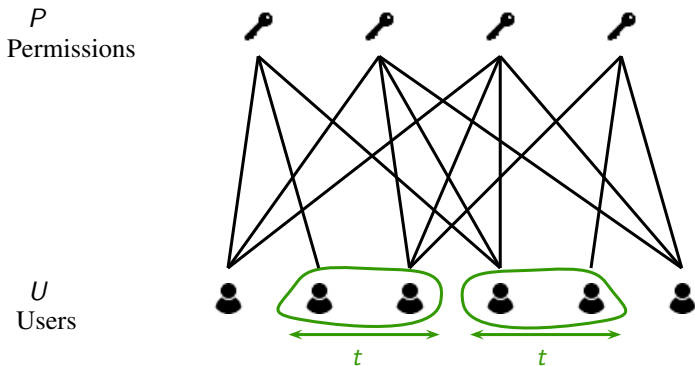
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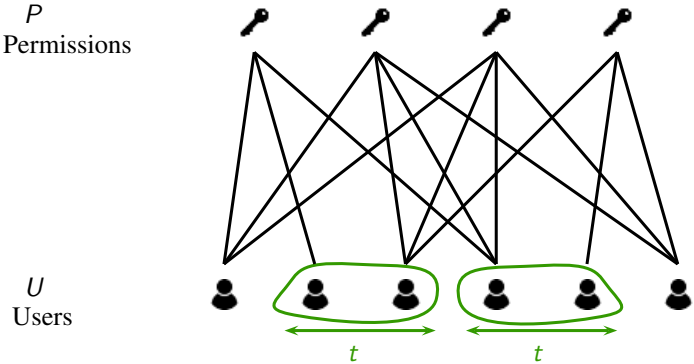
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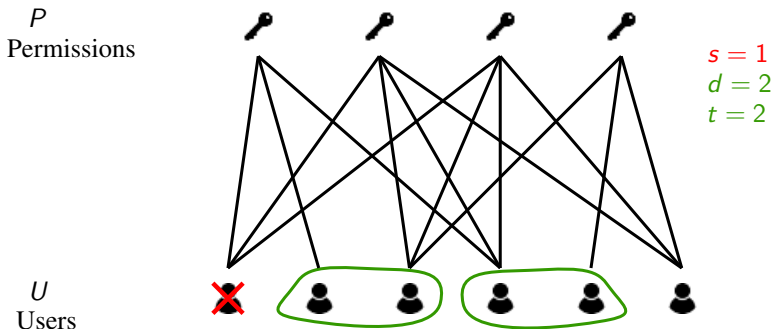
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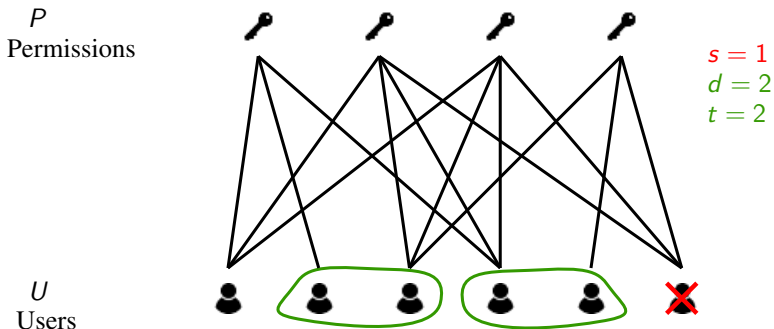
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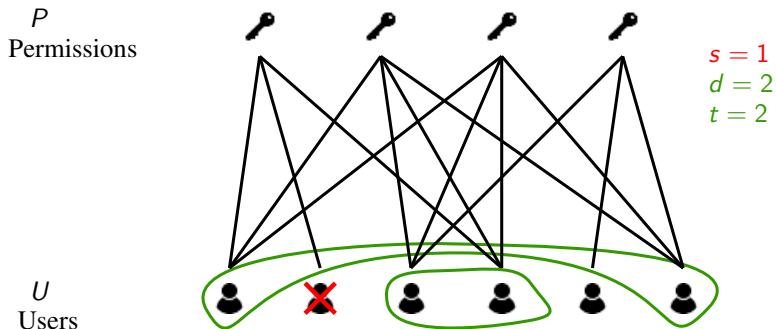
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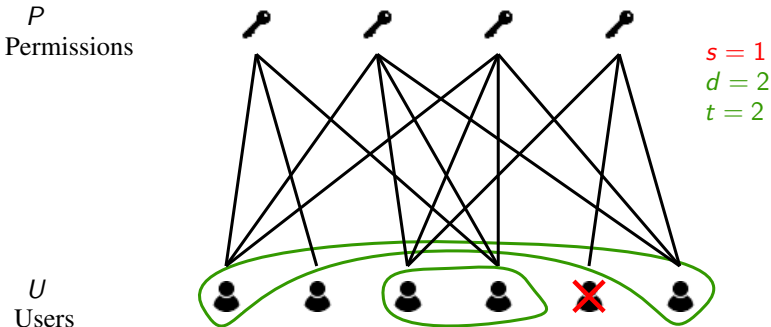


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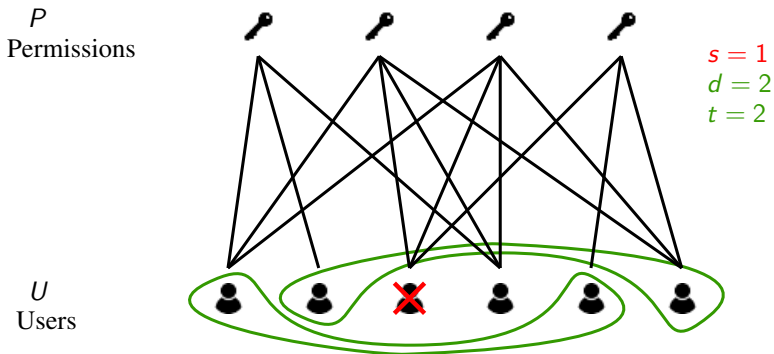
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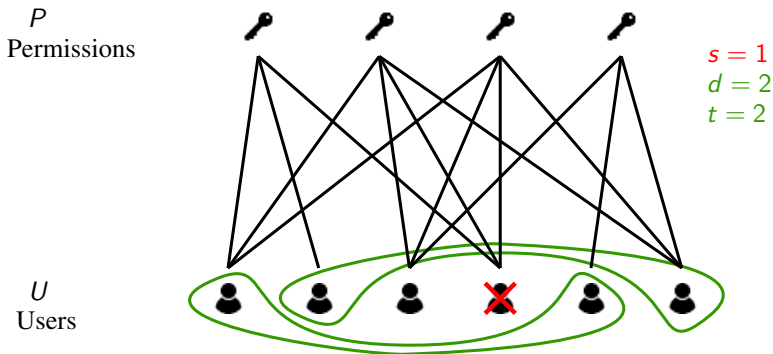
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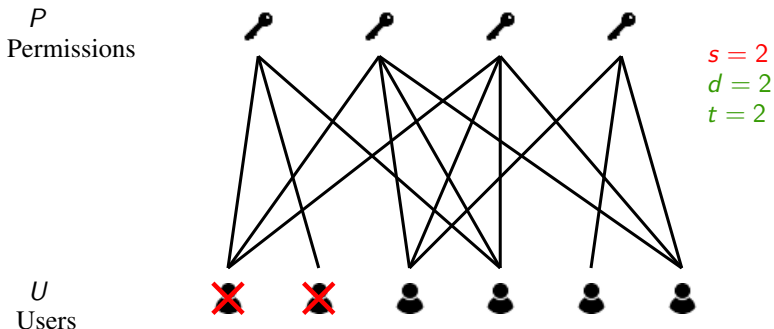
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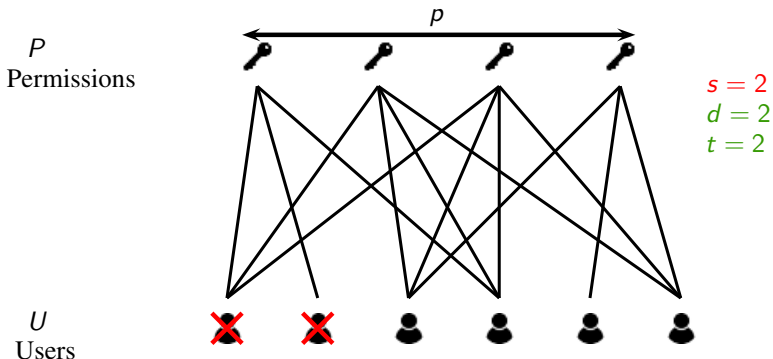
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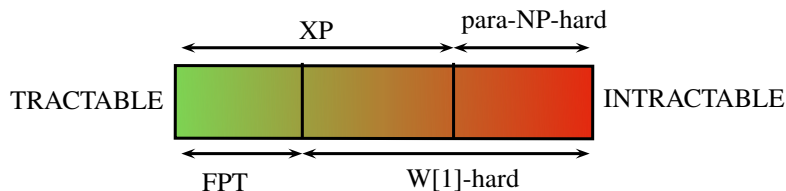
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## Related work

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### Li, Tripunitara, Wang, 2009

- $RCP\langle \rangle, RCP\langle d = 1 \rangle$  and  $RCP\langle t = \infty \rangle$  are NP-hard and in  $coNP^{NP}$ .
- $RCP\langle s = 0, d = 1 \rangle$  and  $RCP\langle s = 0, t = \infty \rangle$  are NP-hard.
- $RCP\langle d = 1, t = \infty \rangle$  is linear-time solvable.

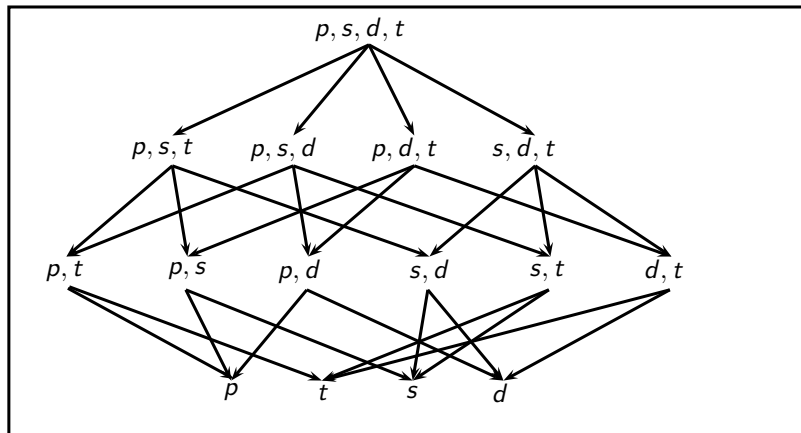
And they present an implementation of an algorithm relying on a SAT formulation.

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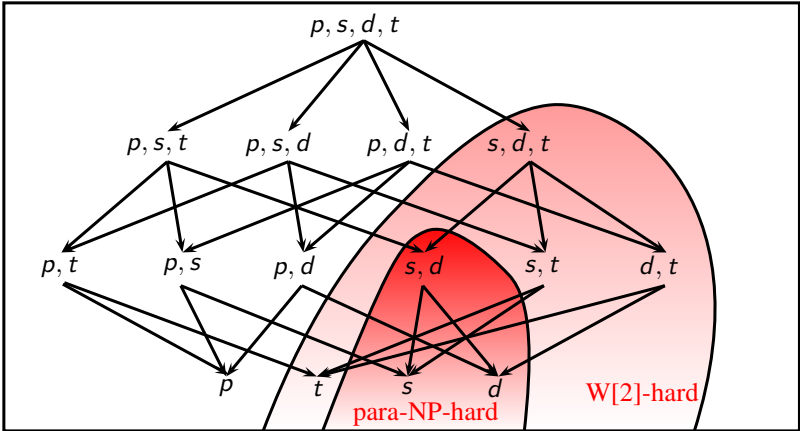
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RCP $\langle s = 0, d = 1 \rangle$  is equivalent to the Hitting Set problem



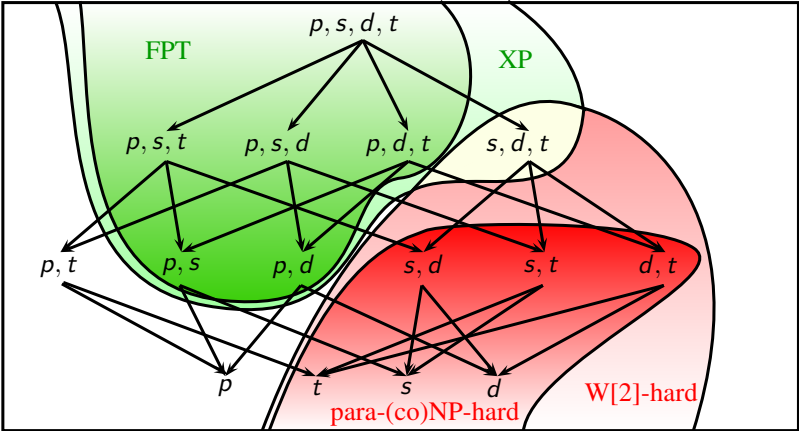
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Results obtained for RCP  $\leftrightarrow$  [Crampton, Gutin, W.]





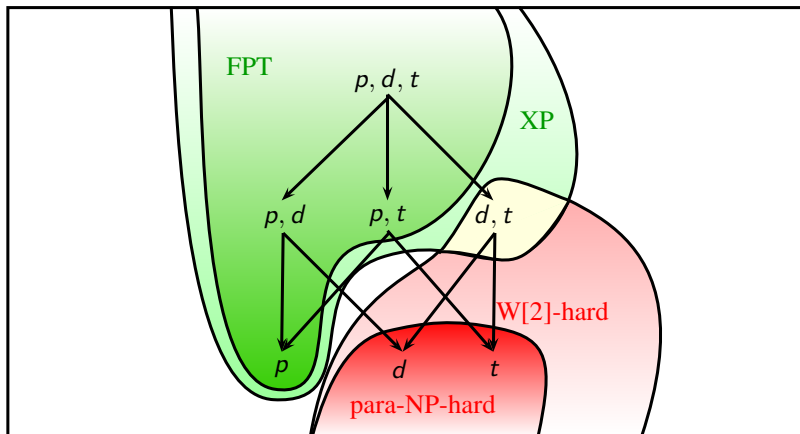
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Results obtained for  $RCP \langle s = 0 \rangle$  [Crampton, Gutin, W.]



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- first: let us show that  $RCP\langle s = 0 \rangle$  is  $FPT$  parameterized by  $p$  only



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Theorem [Lenstra, 1983]+[Kannan, 1987]+[Frank and Tardos, 1987]

Whether a given ILP has a non-empty solution set can be decided in  $O^*(n^{2.5n+o(n)})$  time and polynomial space, where  $n$  is the number of variables.

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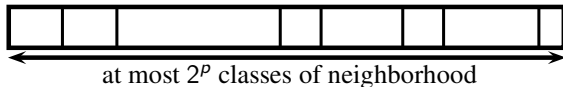


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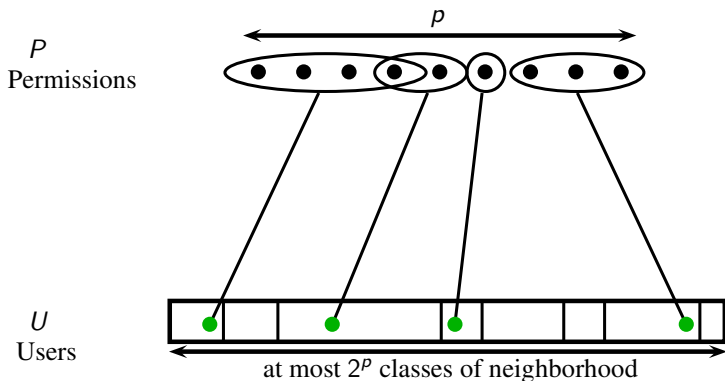


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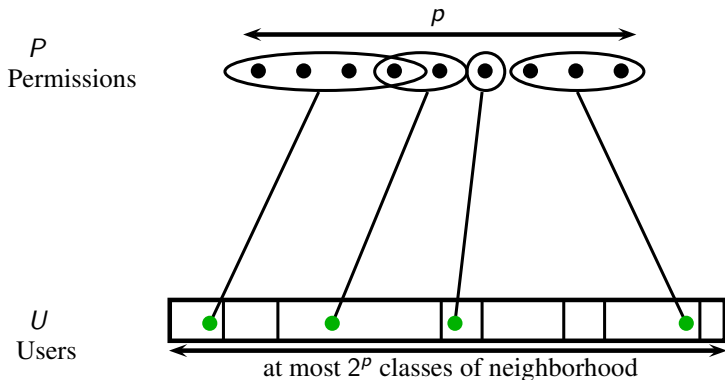
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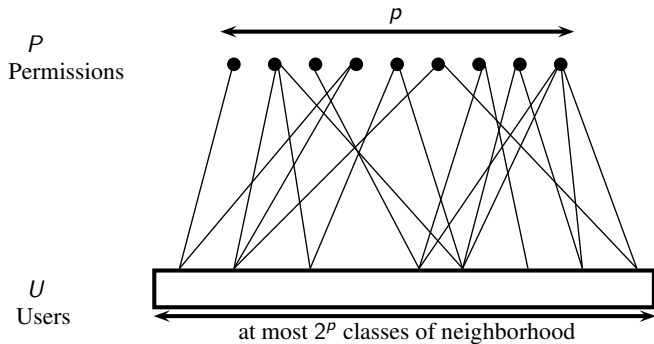
- Second constraint:

$$\sum_{c \in \mathcal{C}[N]} x_c \leq |U[N]| \quad \forall N \subseteq P$$

where:

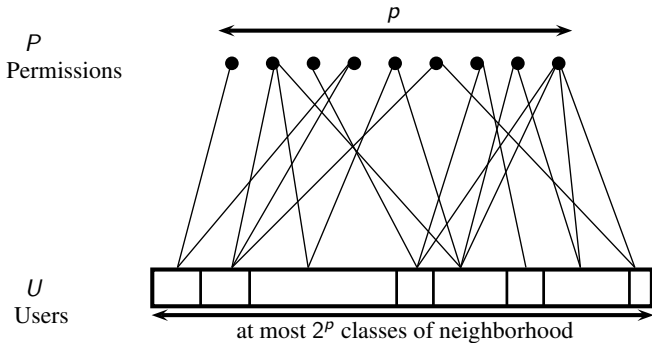
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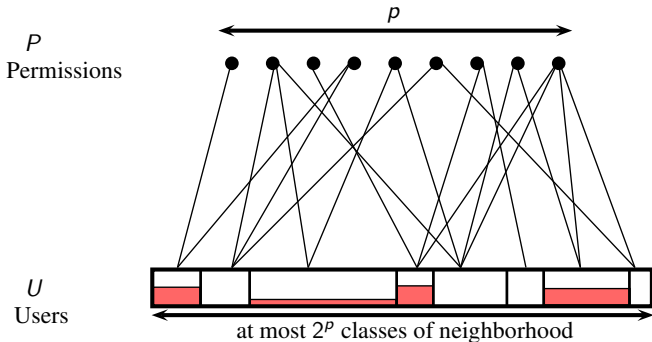
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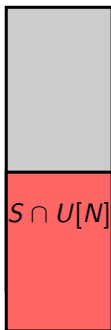


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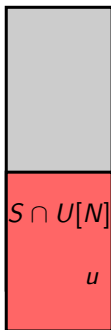
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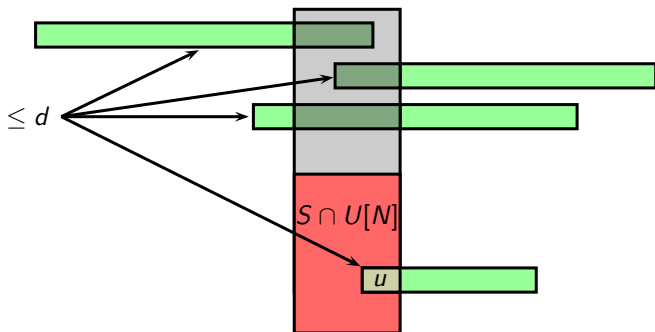
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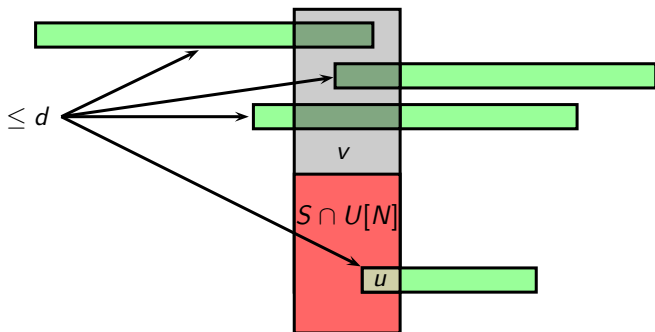


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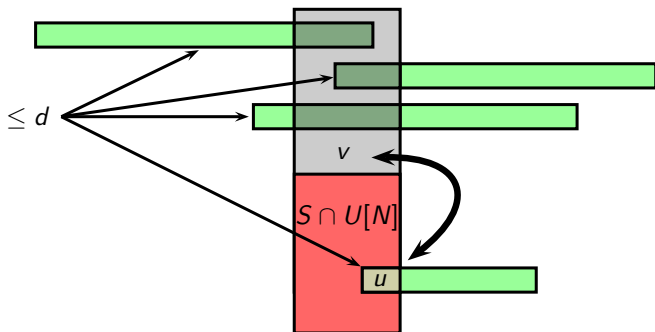
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- Replacing  $u$  by  $v$  creates another set of teams which does not intersect  $S$ :

impossible!



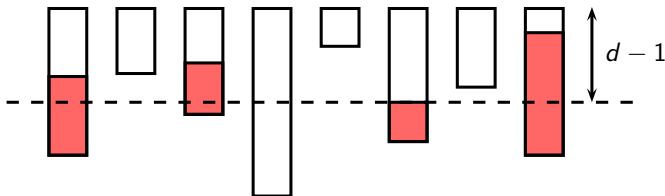
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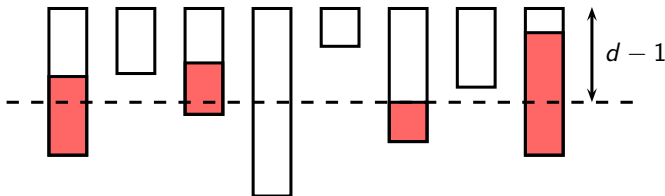


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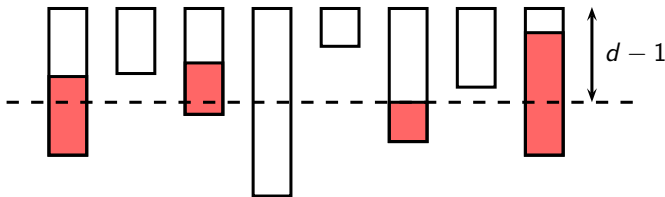


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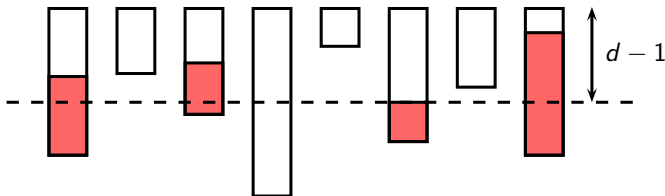


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Open questions:

- what about parameterized by  $p$  only ?
- better running time ? (combinatorial algorithm for RCP $\langle s = 0 \rangle$ )



# Contents

1. Definitions
2. Appetizer: easy observations
3. FPT algorithms
4. Efficient algorithms and implementation
5. Conclusion

## Efficient algorithms?

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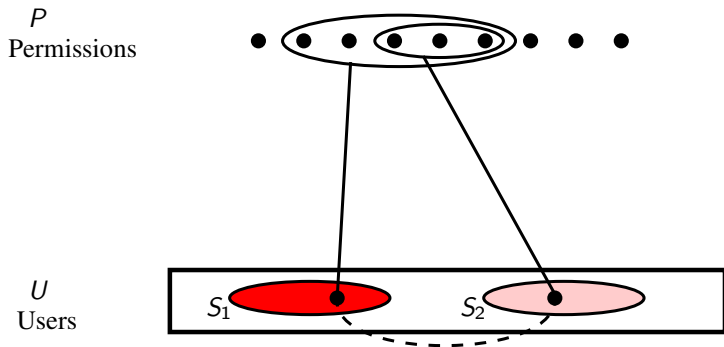
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 $\implies$  what about a fast algorithm for RCP $\langle s = 0 \rangle$  ?
- better not to use:
  - ▶ Lenstra's algorithm/ILP (FPT param. by  $p$  only)
  - ▶ dynamic programming ( $O^*(2^{dp})$  time and space)



# Efficient algorithms?

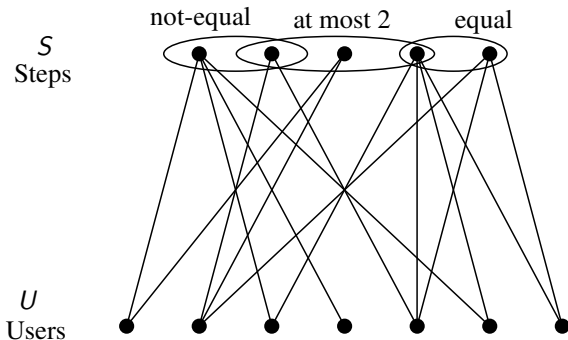
## Workflow Satisfaction Problem (WSP)

Input: a set of steps  $S$ , a set of users  $U$

authorization policy  $A \subseteq U \times S$ , a set of constraints

Output: a plan  $\pi : S \rightarrow U$  such that:

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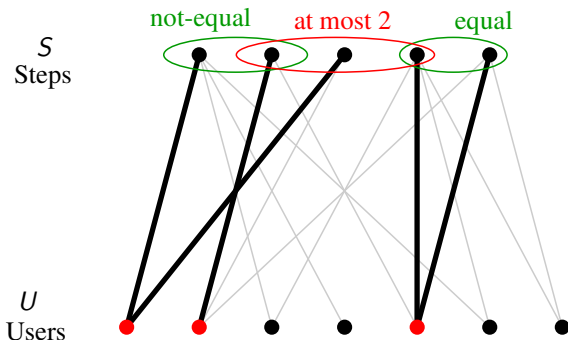
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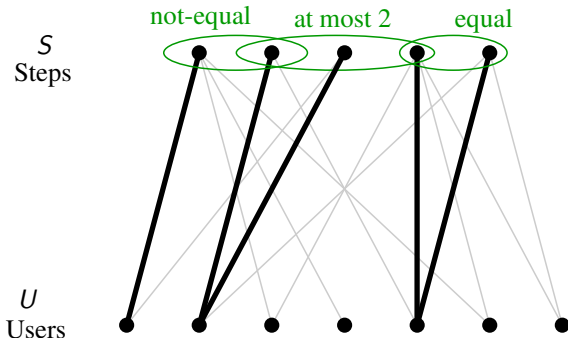
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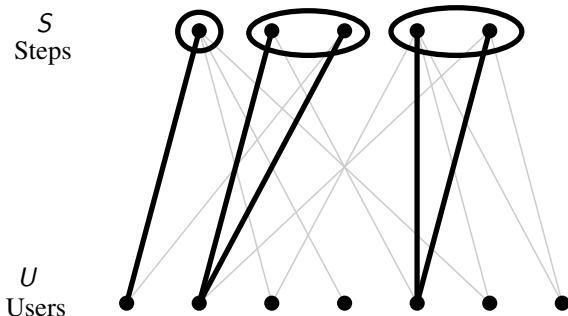
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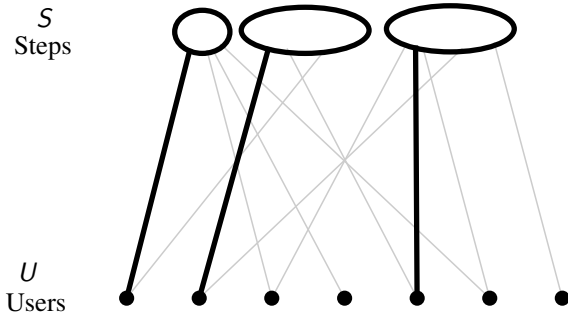
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## Theorem [Karapetyan, Gagarin, Gutin, 2015]

WSP can be solved in  $O^*(2^{k \log k})$ , where  $k$  is the number of steps.

More importantly: an efficient implementation of the algorithm can solve instances with up to 60 steps !

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Theorem [Crampton, Gutin, W.]

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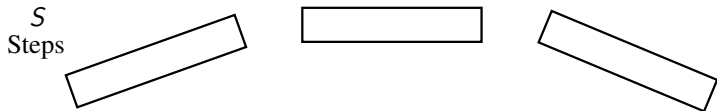


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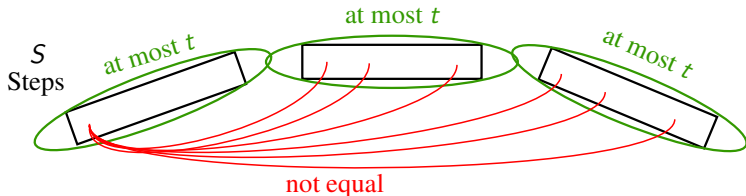


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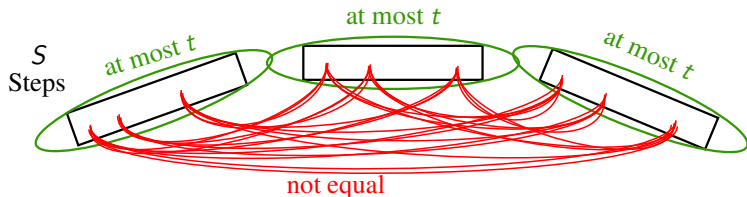


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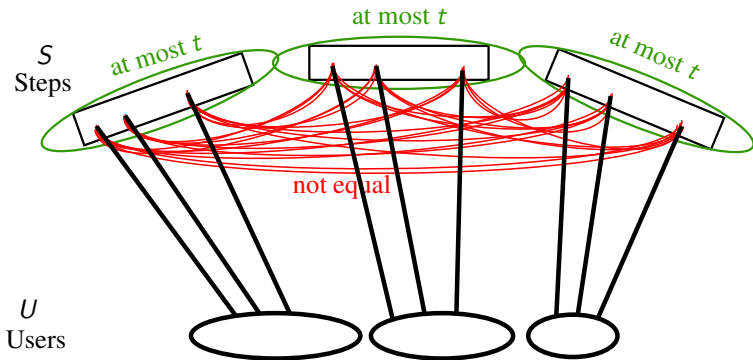


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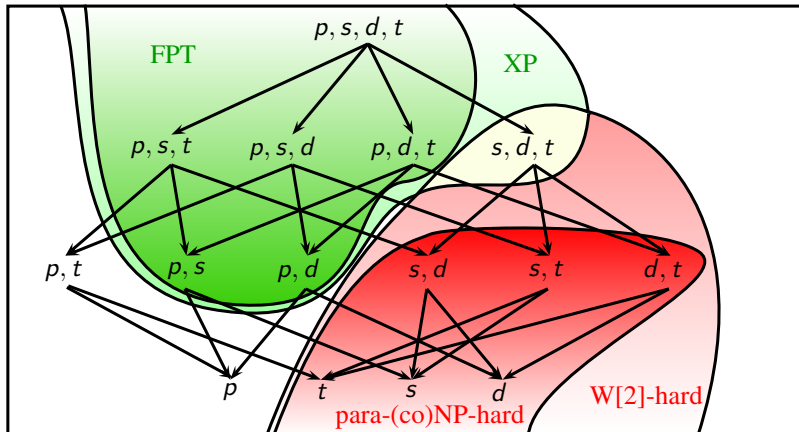
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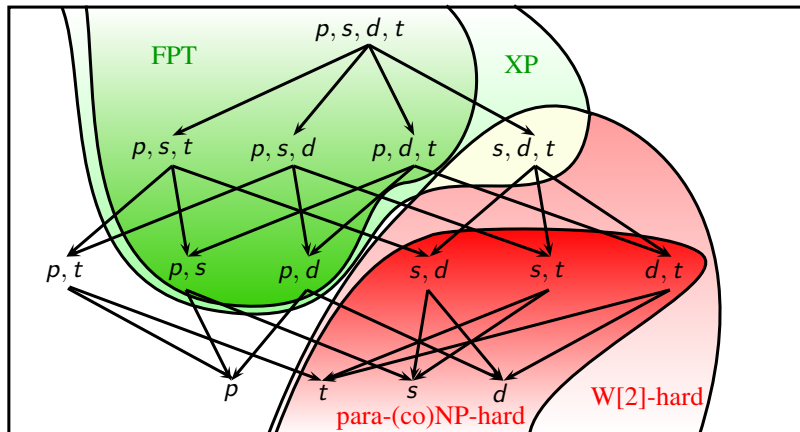
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# Conclusion



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- resiliency w.r.t. other problems ?



Merci !