Monitoring the megathrust seismic cycle using spatial geodesy

Workshop on megathrusts and tsunamis, ICTP, october 2014. marianne.metois@ingv.it

1 Deformation of the lithosphere & seismic cycle

The first observations of crustal deformation around active faults were logically around inland faults for which both sides of the faults are observable. The most famous example are the observations of *Reid* (1910) across the strike-slip San Andreas fault that ruptured during the large 1906 earthquake. Based on offset structures, he proposed that the fault behavior follow an elastic rebound cycle. This intuition gave birth to the seismic cycle theory and to a better understanding of the faults activity. Following this approach, there are two very different deformation periods :

- a long (years to centuries) small (mm/yr) interseismic loading period during which the lithosphere deforms elastically around the fault because of the continuing plate or block motion,
- a rapid (seconds to minutes) large (cm to meters) coseismic deformation accompanying the rupture.

Considering that plates are elastic in a first approximation, ones can predict the deformation pattern produced in both cases. For interseismic period, the fault is considered as locked while the material underneath continues to flow. It creates a typical arctangent shape in the horizontal deformation at the surface given by :

$$V_y = \frac{2V_o}{\Pi}.arctan(\frac{x}{h}),$$

where y is the direction parallel to the strike-slip fault, V_o is the long-term relative plate velocity, and h is the width of locked part of the elastic lithosphere. The deepest goes the locked fault, the widest is the deformation area on the surface. Being able to precisely measure the mm/yr deformation would therefore give constrains on the locking depth and potentially the amount of locking on the fault. The shape of the coseismic deformation is reversed compared to the interseismic one, but the amplitude is much higher and therefore much easy to measure.

One of the fastest strike-slip fault on Earth is the San Andreas fault that is the plate boundary between the Pacific and North American plates. There (as elsewhere in the world) some leveling measurements, triangulation (angle measurements with theodolits) and trilateration (distance measurements with meters and laser recently) campaigns had been conducted since the 70's providing position measurements with great accuracy (<cm). In 1992, when the Landers earthquake ruptured the fault, remeasure of the triangulation network plus some very early GPS measurements enabled to built one of the first coseismic deformation maps *Wald and Heaton* (1994). Dealing with the interseismic deformation, things are more complicated since we are looking locally at some 10mm/yr velocity gradients across the fault (e.g. *Vigny et al.*, 2005), and that even with some good quality triangulation measures we should wait some decades before seeing clearly a signal (in the best case). This is one of the reasons (together with validating the plate tectonics theory and get the current day plate motions) why scientists toke advantage of the new satellite positioning systems that were launched in the 90's.

2 The GPS : principle of GNSS positioning

Some acronyms :

- **GNSS** : Global Navigation Satellite System
- **GPS** : Global Positioning System (USA)
- **Galileo** : European positioning system
- **GLONASS** : Russian positioning system
- **IRNSS** : Indian Regional Navigational Satellite System
- **Compass** : BeiDou Satellite Navigation System (China)

The motivation of GNSS system is to provide the users with their position instantaneously wherever they are on Earth. Initially, the GPS system was launch for military purposes by the US army and made gradually available for public use in after 1995. The principle is exactly the same than for triangulation : if you know the distance between you and at least three satellites (which position is known) at a given time you are able to determine your position. First, to see 3 satellites from every point on Earth at every time, you need a constellation of at least 24 satellites in 6 different orbits at 20.200 km high. The recent fails in launching the Galileo satellite remind us that making this constellation is already not an easy thing. Second, how can we calculate the distance satellite-station ?

The GPS satellites (forming part of the spatial segment of the system) send several signals that are received by the antenna :

- **Carrier signals/sinusoides** : L1 (frequency 1.575 GHz) and L2 (frequency 1.228 GHz) ultra-high frequency signals (passing through clouds)
- **Codes** : binary signals P/Y (Precise-Y code affected by antispoofing by US army up to 2000), and C/A (Coarse Acquisition) codes are supported by both L1 and L2 carriers.

The antenna and receiver forming the user segments generate an inner copy of the signals. Comparing this clone signal to the one received gives access to the travel time of the signal between the satellite and the station, so, potentially to the distance satellite-station. The codes signals are unambiguous since their repeating time is high (λ of 3m and 30m, so the repeating time is much higher than the $\sim$5ms between satellite and station), but they are limited to a precision of tens of meters (for C/A) or cm (for P/Y if no antispoofing was applied). However, they are used for the calculation of **"pseudo-distances"** in commercial GPS in which no further processing or correction is applied. Therefore, your palmer GPS or car GPS gives you a position precise at 10m (careful!).

This is well above what would be of use for scientific purposes (we are aiming at measuring mm/yr velocities!). But the carrier signals are of help since they could potentially offer mm accuracy for the position (fraction of their wavelength). The phase of the signal received by the station from the satellite at a given time t is given by

$$\Phi_S(t) = f.t - \frac{f.d}{v}$$

where f is the frequency, d the distance satellite-station and v the wave velocity. The receiver inner oscillator generates a signal whose phase is simply $\Phi_R(t) = f.t$. Therefore, the phase shift between both signals is given by

$$\Delta\Phi_{sr}(t) = \Phi_R(t) - \Phi_S(t) = \frac{f.d}{v}$$

In fact, since $d >> \lambda$, the carrier signals are ambiguous and we have

$$d = (\Delta\Phi_{sr}(t) + N)\frac{v}{f}$$

where N is an unknown number of cycles called ambiguity. The receiver calculates the phase shift $\Delta\Phi_{sr}$ between the received signal and the inner signal but it is unable to determine directly the exact number N. Furthermore, all along the satellite-station path, the signal is affected (accelerated, slowed down, reflected) by the inhomogeneous medium crossed. Therefore,

$$d = (\Delta\Phi_{sr}(t) - \Phi_{noise} - \Phi_{clocks} - \Phi_{iono} - \Phi_{tropo} + N).\frac{v}{f}$$

Last but not least, knowing the exact position and time of the satellite when sending the signal requires further calculations and corrections. Finally, only a careful processing of the GPS signal could lead to know the position of a point with a several mm accuracy, but this is well beyond the scope of instantaneous positioning.

2.1 Issue 1 : the clock drifts

To correctly measure $\Delta\Phi_{sr}$, we need to correct for the clock drifts of both the satellite and receiver oscillators that produce the real signal and the inner receiver copy respectively. The satellite clock is an atomic clock with a small drift of 10^{-9}s/day that is regularly detected and corrected by the control segment (i.e. Doris stations for instance that are permanently in touch with the constellation). The receiver clock is a commercial low-quality clock with an average drift of 10^{-5} to 10^{-6}s/day.

With $v \sim 3.10^8$m/s and an average distance satellite-station of 20.000km, the standard duration of the satellite-station path is around 70 ms. A change of 1 meter in the satellite-station distance would lead to a change in the path duration of $\sim 3.10^9$ seconds. Correcting for the drifts of the receiver and satellite clocks is therefore highly necessary. For a full mathematical expression of the drift-induced delays, refer to *King et al.* (1985) or *Hofmann-Wellenhof et al.* (1993). Let's consider the phase measurement between the satellite S_1 and the receiver R_1 as :

$$\Phi_{measured_{S_1R_1}} = \Delta\Phi_{S_1R_1} - d\Phi_{clocks_{S_1}} - d\Phi_{clok_{R_1}} + N$$

while the phase measurement between the satellite S_2 and the same receiver will be :

$$\Phi_{measured_{S_2R_1}} = \Delta\Phi_{S_2R_1} - d\Phi_{clocks_{S_2}} - d\Phi_{clok_{R_1}} + N$$

Therefore, a simple combination of both measurements, called **"simple difference"**, will cancel the delays associated to the receiver clock :

$$\Phi_{m_{S_2R_1}} - \Phi_{m_{S_1R_1}} = \Delta\Phi_{S_2R_1} - \Delta\Phi_{S_1R_1} - d\Phi_{clocks_{S_2}} + d\Phi_{clok_{S_1}} + N$$

To get rid off the satellite clock drifts, one may consider to combine all the phase measurements from 2 satellites and 2 receivers in a **"double difference"** phase measurement :

$$(\Phi_{m_{S_2R_1}} - \Phi_{m_{S_1R_1}}) - (\Phi_{m_{S_2R_2}} - \Phi_{m_{S_1R_2}}) = \Delta\Phi_{S_2R_1} - \Delta\Phi_{S_1R_1} - \Delta\Phi_{S_2R_2} + \Delta\Phi_{S_1R_2} + N$$

But this last combination will lead to the calculation of the baseline between R_1 and R_2 rather than to the position calculation. Therefore, this double difference calculation is called differential positioning and implies to process all the stations of the network together, that is quite time consuming. An alternative to the double difference is to use the simple-difference phase measurements together with the corrections given by JPL for the satellite clocks. In this latter case, the processing is called **PPP** for precise point positioning and gives an absolute position for each station independently.

2.2 Issue 2 : the satellite position

The publicly available orbits delivered by the US Army for the GPS satellites are uncertain to $\sim$ 200m, i.e. $\sim 10^{-5}$ the distance satellite-receiver we wish to calculate. In a "double-difference" calculation, it will produce a 10 cm uncertainty on a 10 km baseline. A posteriori, one can recalculate the satellite's orbits with tens of centimeter precision therefore reducing to $\sim$1mm the uncertainty on the baseline measurement. Using such reprocessed orbits (for instance from IGS) improves greatly the position calculation (*Beutler et al.*, 1999).

2.3 Issue 3 : the ionospheric delay

Between the satellite and the station, the GPS signal crosses the ionosphere (i.e ionized part of the atmosphere above $\sim$60km) that is a diffracting medium that affects the wave velocity depending on its frequency f and on the density in electrons of the ionosphere, named K. The delay dt induced by crossing the ionosphere is given by

$$dt\alpha - K/f^2,$$

thus because we can express the phase Φ as

$$\Phi(t) = f.(t - \frac{d}{v})$$

where f is the frequency, d the distance satellite-station and v the wave velocity, the associated phase shift goes with

$$\Delta\Phi_{iono}\alpha - K/f.$$

Therefore, combining both L1 and L2 phase measurements in the "LC" combination can cancel the ionospheric delay impact :

$$\Phi_{measured_{L_1}} = \Delta\Phi_{L_1} - K/f_1 + N$$

$$\Phi_{measured_{L_2}} = \Delta\Phi_{L_2} - K/f_2 + N$$

$$\Phi_{LC} = \frac{f_1^2}{f_1^2 - f_2^2}(\Phi_{m_{L_1}} - \frac{f_2}{f_1}\Phi_{m_{L_2}}) = \frac{f_1^2}{f_1^2 - f_2^2}(\Delta\Phi_{L_1} - \frac{f_2}{f_1}\Delta\Phi_{L_2} + N)$$

2.4 Issue 3 : the tropospheric delay

This is certainly the most complicated delay to model or estimate. It is due to the fact that the troposphere (i) changes in width and (ii) contains water vapor that slow down the GPS waves. Usually, we consider to kind of delays associated to the path through the troposphere : the dry delay due to the width of the troposphere itself and location of the satellite, and the wet delay due to the water vapor content. The zenital dry delay is expressed as

$$ZTD_{dry} = 1.013x2.27e^{0.000116h},$$

where h is the width of the troposphere above the station. We can estimate h using the laws of the ideal gases if the GPS station is monitored with pressure and temperature sensors, which is rarely the case. In general, we use some hydrostatic **mapping functions** of the troposphere (e.g. *Saastamoinen*, 1972; *Niell*, 1996; *Simmons et al.*, 2007) and correct from the elevation angle of the satellite. To avoid using signals that crossed a very wide and noisy troposphere, satellite with an elevation angle relative to the station lower than 10° are discarded.

The wet delay can be also computed from wet mapping functions but is in general inverted every 2 hours or even more in the processing together with some horizontal gradients. Bad estimates of the tropospheric delay (because of local meteorological instabilities or specific sites located near the sea for instance) will produce large errors mainly on the vertical position. Indeed, errors on the horizontal position will be averaged in some way if measurements are conducted on an entire day or more because of the satellite trajectory, that could not be the case for vertical position since signal is always coming from above. GPS located in area that are supposed not to be deforming are often used to monitor the atmosphere and water content.

2.5 Issue 4 : the multipath effect

We have seen that combining phase measurements can significantly help removing some delays due to the nature of the medium in which the wave propagates. However, we have also to pay attention to geometric issues mainly due to reflectors. Indeed : how can we be sure that the signal received by the receiver is coming directly from the satellite ? There is no way to do that, except by carefully selecting sites away from any probable reflectors (mountains, house, etc), but even the soil can be a reflector. Therefore, one has to keep in mind that the calculated position is not the one of the antenna but probably the one of an image of the antenna relative to the reflector. Long measurements will produce an averaged position that is close to the real antenna position because of the satellite motion in the sky.

2.6 Issue 5 : the antenna phase-center drift

The GPS signal is transformed into electric signal through a coil located in the antenna. The antenna phase-center position we finally calculate through processing depends on the incidence angle of the signal relative to the coil. Since satellite is moving, the phase-center also moves and some centimetric variations can be observed. This is again a limitation to the use of short measurement sessions to precisely compute the position. Several hours and days are necessary to average this variation. However, if the antenna has been extremely well calibrated on the lab some post-processing corrections can be done... if you know exactly the antenna orientation relative to the North !

2.7 Issue 6 : estimating the ambiguities

Finally, even correcting from all the delays and reflections of the signal, the carrier waves remain ambiguous to a number N of phase cycles. N is time-independent if the contact between the satellite and the receiver is continuous. The basic idea of the ambiguity inversion is to try to combine all the available phase and code observations to get a set of equations from which a least-square algorithm could be applied to both the distance and ambiguity value. This is much better to solve for ambiguities for double-differenced combinations since you get rid of the receiver clock errors. If N is forced to be integer, the ambiguities are said "fixed" or "resolved", on the contrary, if N can be fraction of the wavelength, ambiguities are said to be "free". Usually, the "fixed" ambiguities lead to much higher precision.

To understand how we could estimate the ambiguities, lets analyze the combination used for solving ambiguities based on both the phase and code measurements. Lets go back to the expression for the phase Φ that is in fact more complex than what has been presented before:

$$\Phi_S(t) = f.t - \frac{f.d}{v} + \Phi_S(t_o)$$

$$\Phi_R(t) = f.t + \Phi_R(t_o)$$

where $\Phi_R(t_o)$ and $\Phi_S(t_o)$ are the phases delays due to the receiver and satellite clock shifts at t_o, that are directly proportional to the frequency, as $f\delta$. Therefore,

$$\Phi_{SR}(t) = \frac{fd}{v} + f.\Delta\delta + N$$

Because of the diffracting effect of the ionosphere introducing a $\alpha 1/f^2$ delay, we can write :

$$\Phi_{SR}(t) = \frac{fd}{v} + f.\Delta\delta + N - \frac{K}{f}$$

$$\Phi_{SR}(t) = a.f + N - \frac{b}{f}$$

Where a is a geometric term and b the ionospheric term. This implies for both carrier waves :

$$\Phi_1 = a.f_1 + N_1 - \frac{b}{f_1}$$

$$\Phi_2 = a.f_2 + N_2 - \frac{b}{f_2}$$

One way to solve for the ambiguities is to consider linear combinations between the phases of the L1 and L2 carriers, as the **Wide Lane** combination given by

$$\Phi_{WL} = \Phi_1 - \Phi_2$$

that leads to

$$\Phi_{1-2} = a.f_{1-2} + N_{1-2} - \frac{b}{f_{1-2}}$$

where N_{1-2} is the Wide-Lane ambiguity. The wave-length associated to the WL combination is of 86.2 cm, i.e. much higher than λ_1 and λ_2, that will make the solving of the WL ambiguity N_{1-2} much easier, even without using the code signals. Recombining, you will find

$$N_1 = \Phi_1 - \frac{f_1}{f_{1-2}}.(\Phi_{1-2} - N_{1-2}) + b.\frac{f_1 + f_2}{f_1.f_2}$$

Now, if we use the code ranges R on the L_1 or L_2 carriers, we have

$$R_1 = af_1 + \frac{b}{f_1}$$

$$R_2 = af_2 + \frac{b}{f_2}$$

Simple combinations with expressions of Φ_1 and Φ_2 give the expression for the Wide-Lane ambiguity without the geometric and ionospheric factors.

$$N_{1-2} = \Phi_{1-2} - \frac{f_1 - f_2}{f_1 + f_2}.(R_1 + R_2)$$

References

Beutler, G., M. Rothacher, and S. Schaer (1999), The International GPS Service (IGS): an interdisciplinary service in support of earth sciences, *Advances in Space*, *23*(4), 631–653.

Hofmann-Wellenhof, B., H. Lichtenegger, and J. Collins (1993), Global positioning system. theory and practice., *Global Positioning System. Theory and practice., by Hofmann-Wellenhof, B.; Lichtenegger, H.; Collins, J.. Springer, Wien (Austria), 1993, 347 p., ISBN 3-211-82477-4, Price DM 79.00. ISBN 0-387-82477-4 (USA).*, *1*.

King, R. W., E. Masters, C. Rizos, A. Stolz, and J. Collins (1985), *Surveying with GPS*, School of Surveying, the University of New South Wales.

Niell, A. (1996), Global mapping functions for the atmosphere delay at radio wavelengths, *Journal of Geophysical Research*, *101*, 3227–3246.

Reid, H. (1910), *The California Earthquake of April 18, 1906: The Mechanics of the Earthquake/By Harry Fielding Reid*, Carnegie Inst.

Saastamoinen, J. (1972), Atmospheric correction for the troposphere and stratosphere in radio ranging satellites, *Geophysical Monograph Series*, *15*, 247–251.

Simmons, A., S. Uppala, D. Dee, and S. Kobayashi (2007), ERA-Interim: New ECMWF reanalysis products from 1989 onwards, *ECMWF newsletter*, *110*, 25–35.

Vigny, C., et al. (2005), Insight into the 2004 sumatra–andaman earthquake from gps measurements in southeast asia, *Nature*, *436*(7048), 201–206.

Wald, D. J., and T. H. Heaton (1994), Spatial and temporal distribution of slip for the 1992 landers, california, earthquake, *Bulletin of the Seismological Society of America*, *84*(3), 668–691.